Appendix

Proof of Proposition 1. For any price p within the support of its pricing strategy, seller i's expected profit is

$$\pi_i(p,\alpha_i) = E\big(R_i(p,\alpha_i)\big) - C(\alpha_i) = p\alpha_i \cdot E\left\{\prod_{j\neq i}[1-\alpha_j F_j(p)]\right\} - C(\alpha_i). \qquad \text{(A1)}$$

Given any strategies of the other sellers, α_j and $F_j(p)$, $j \neq i$, the first term on the right-hand side (expected revenue) is linear in α_i, but the second term (cost) is strictly convex in α_i. Hence, the sum is strictly concave in α_i, which means that seller i can maximize profit at some $\alpha_i^* > 0$. Accordingly, seller i will not randomize acquisitions. [ ]

Proof of Proposition 2. By (1) and (2), the profit of seller $i = 1, \ldots, n$, at any price p within the support of its pricing strategy, is

$$\pi_i(\alpha_i, F_i) = p\alpha_i \prod_{j\neq i}[1-\alpha_j F_j(p)] - C(\alpha_i). \qquad \text{(A2)}$$

Since sellers acquire consumers and set prices simultaneously, the other sellers' pricing strategies, $F_j(\cdot)$, do not depend on seller i's acquisitions, α_i. Hence, the profit maximization problem of seller i is globally concave.

Now, in a randomized-strategy equilibrium, seller i must earn the same profit for all $p \in [\underline{p}, v]$. In particular, by Lemma 1, at $p = \underline{p}$, $F_j(\underline{p}) = 0$ for all $j = 1, \ldots, n$. Substituting into (A2), seller i's profit simplifies to

$$\pi_i(\alpha_i, F_i) = \underline{p}\alpha_i - C(\alpha_i),$$

and the solution is characterized by the first-order condition

$$\underline{p} = C'(\alpha_i). \qquad \text{(A3)}$$

For any other seller l, we also have

$$\underline{p} = C'(\alpha_l). \qquad \text{(A4)}$$

Conditions (A3) and (A4) are consistent only if $\alpha_i = \alpha_l = \alpha_s$, say. Hence, equilibrium acquisitions are symmetric. By Lemma 1, when acquisitions are symmetric, the pricing strategies do not include any mass points. Hence, at $p = v$, $F_j(v) = 1$ for all j = 1, …, n. Since the revenue of every seller at $p = \underline{p}$ and $p = v$ must be the same in a randomized-strategy equilibrium, and the cost of customer acquisition does not depend on p, seller i's revenue is

$$p\alpha_i \prod_{j \neq i} [1 - \alpha_j F_j(p)] = v\alpha_i [1 - \alpha_s]^{n-1} = \underline{p}\alpha_i . \tag{A5}$$

After substituting from (A3), we have (5).

Now, by Lemma 1, since acquisitions are symmetric, the pricing strategies are also symmetric, with $F_j = F_s$, say, for all j = 1, …, n. Substituting $\alpha_j = \alpha_s$ and $F_j = F_s$ into the first equality of (A5), we have

$$[1 - \alpha_s F_s(p)]^{n-1} = \frac{v}{p}[1 - \alpha_s]^{n-1}, \tag{A6}$$

which simplifies to (6). Finally, at $p = \underline{p}$, $F_s(\underline{p}) = 0$. Substituting this into (6) yields (7).

Finally, we need to show that sellers would not deviate from this equilibrium. In equilibrium, by (A2) and (A5), for any seller i = 1, …, n,

$$\pi_i(\alpha_s, F_s) = v\alpha_s[1 - \alpha_s]^{n-1} - C(\alpha_s) \geq v\alpha_i[1 - \alpha_s]^{n-1} - C(\alpha_i) \tag{A7}$$

for all α_i. By (7), (A7) simplifies to

$$\pi_i(\alpha_s, F_s) = \underline{p}\alpha_s - C(\alpha_s) \geq \underline{p}\alpha_i - C(\alpha_i) . \tag{A8}$$

We first consider a deterministic deviation to some price p and acquisition α_i', and show that such a deviation will not raise profit. Specifically, suppose that seller i deviates to a strategy (α_i', p), where $\alpha_i' \neq \alpha_s$.

Case (i): $p < \underline{p}$. In this case, seller i's price is lower than all other sellers. So it would sell to every consumer that it acquires; hence, its expected profit from the deviation would be:

$$\pi_i(\alpha_i', p) = p\alpha_i' - C(\alpha_i') < \underline{p}\alpha_i' - C(\alpha_i') \leq \underline{p}\alpha_s - C(\alpha_s) = \pi_i(\alpha_s, F_s), \qquad \text{(A9)}$$

where the first inequality is trivial because $p < \underline{p}$, and the second inequality is implied by (A8). Hence, the seller will not deviate.

Case (ii): $\underline{p} \leq p \leq v$. In this case, seller i sells to those consumers who are not acquired by any other sellers, and those who are acquired by other sellers if seller i's price is the lowest. Substituting (A6) into (A2), seller i's expected profit from the deviation would be

$$\begin{aligned}\pi_i(\alpha_i', p) &= p\alpha_i' \cdot \frac{v}{p}[1-\alpha_s]^{n-1} - C(\alpha_i') \\ &= v\alpha_i'[1-\alpha_s]^{n-1} - C(\alpha_i') \\ &\leq v\alpha_s[1-\alpha_s]^{n-1} - C(\alpha_s) = \pi_i(\alpha_s, F_s),\end{aligned}$$

where the inequality is implied by (A7). Hence, the seller would also not deviate.

Finally, the profit from deviation to any randomized pricing strategy, (α_i', F_i), cannot exceed the expectation over the profits from the various (α_i', p) with deterministic prices. Since deviating to each (α_i', p) does not raise profit, the deviation to a randomized pricing strategy cannot increase profit too. Hence, no seller will deviate. [ ]

Customer Acquisition before Pricing: Two-Seller Symmetric Equilibrium

Expanding (12),

$$
\begin{aligned}
\pi(\alpha_i) &= \left\{ \int_{\alpha_j>\alpha_i} [1-\alpha_i] dG(\alpha_j) + \int_0^{\alpha_i} [1-\alpha_j] dG(\alpha_j) \right\} v\alpha_i - C(\alpha_i) \\
&= \left[\int_{\alpha_j>\alpha_i} dG(\alpha_j) - \alpha_i \int_{\alpha_j>\alpha_i} dG(\alpha_j) + \int_0^{\alpha_i} dG(\alpha_j) - \int_0^{\alpha_i} \alpha_j dG(\alpha_j) \right] v\alpha_i - C(\alpha_i) \\
&= \left\{ 1 - \alpha_i \left[1 - \int_0^{\alpha_i} dG(\alpha_j) \right] - \int_0^{\alpha_i} \alpha_j dG(\alpha_j) \right\} v\alpha_i - C(\alpha_i) \\
&= \left\{ 1 - \alpha_i + \int_0^{\alpha_i} [\alpha_i - \alpha_j] dG(\alpha_j) \right\} v\alpha_i - C(\alpha_i).
\end{aligned} \tag{A10}
$$

Substitute $s \equiv \alpha_j$, and then, using integration by parts, the integral on the right-hand side of (A10) simplifies as follows,

$$
\begin{aligned}
\int_0^{\alpha_i} [\alpha_i - \alpha_j] dG(\alpha_j) &= \int_0^{\alpha_i} [\alpha_i - s] dG(s) = \alpha_i \int_0^{\alpha_i} dG(s) - \int_0^{\alpha_i} s dG(s) \\
&= \alpha_i [G(s)]_0^{\alpha_i} - \left\{ [sG(s)]_0^{\alpha_i} - \int_0^{\alpha_i} G(s) ds \right\} = \int_0^{\alpha_i} G(s) ds.
\end{aligned} \tag{A11}
$$

Substituting (A11) in (A10), then (12) simplifies to

$$
\pi(\alpha_i) = v\alpha_i[1-\alpha_i] + v\alpha_i \int_0^{\alpha_i} G(s) ds - C(\alpha_i). \tag{A12}
$$

The profit in (A12) must be constant over the support of the randomized strategy. At the lower bound of the distribution, $G(\underline{\alpha}) = 0$, and so, by (A12),

$$
\pi(\underline{\alpha}) = v\underline{\alpha}[1-\underline{\alpha}] + v\underline{\alpha} \int_0^{\underline{\alpha}} G(s) ds - C(\underline{\alpha}) = v\underline{\alpha}[1-\underline{\alpha}] - C(\underline{\alpha}). \tag{A13}
$$

For the randomized strategy to be an equilibrium, no $\alpha_i < \underline{\alpha}$ may yield higher profit. Accordingly, the lower bound, $\underline{\alpha}$, is defined by

$$
\underline{\alpha} = \arg\max \{ v\alpha[1-\alpha] - C(\alpha) \}. \tag{A14}
$$

Differentiating (A12) with respect to α_i, for all $\alpha_i \geq \underline{\alpha}$, the function $G(\cdot)$ satisfies the differential equation,

$$\frac{d\pi}{d\alpha_i} = v[1-2\alpha_i] + v\int_0^{\alpha_i} G(s)ds + v\alpha_i \frac{d}{d\alpha_i}\left(\int_0^{\alpha_i} G(s)ds\right) - C'(\alpha_i) = 0, \tag{A15}$$

which further simplifies to

$$v\int_0^{\alpha_i} G(s)ds + v\alpha_i G(\alpha_i) = C'(\alpha_i) - v[1-2\alpha_i]. \tag{A16}$$

Differentiating (A16) again, we have

$$vG(\alpha_i) + vG(\alpha_i) + v\alpha_i G'(\alpha_i) = C''(\alpha_i) + 2v.$$

Simplifying and multiplying throughout by the integrating factor, α_i,

$$2v\alpha_i G(\alpha_i) + v\alpha_i^2 G'(\alpha_i) = \alpha_i C''(\alpha_i) + 2v\alpha_i.$$

Integrating with respect to α_i,

$$\begin{aligned} v[\alpha_i^2 G(\alpha_i)] &= \int \alpha_i C''(\alpha_i) d\alpha_i + 2v\int \alpha_i d\alpha_i + K \\ &= [\alpha_i C'(\alpha_i) - C(\alpha_i)] + v\alpha_i^2 + K, \end{aligned} \tag{A17}$$

where K is an integration constant.

Substituting the boundary condition, $G(\underline{\alpha}) = 0$, in (A16),

$$0 = C'(\underline{\alpha}) - v[1-2\underline{\alpha}], \tag{A18}$$

and also substituting in (A17),

$$0 = [\underline{\alpha}C'(\underline{\alpha}) - C(\underline{\alpha})] + v\underline{\alpha}^2 + K. \tag{A19}$$

Substituting from (A18) and (A19) in (A13), we have

$$K = -\pi(\underline{\alpha}). \tag{A20}$$

Substituting (A20) into (A17), and rearranging terms,

$$v\alpha_i^2[1-G(\alpha_i)] = \pi(\underline{\alpha}) - [\alpha_i C'(\alpha_i) - C(\alpha_i)]. \tag{13}$$

Substituting the boundary condition, $G(\hat{\alpha}) = 1$, in (A17), and using (A20), the upper limit, $\hat{\alpha}$, is defined by

$$\hat{\alpha}C'(\hat{\alpha}) - C(\hat{\alpha}) = -K = \pi(\underline{\alpha}). \tag{A21}$$

By (A19), (A21) and (13), it is easy to see that $G(\alpha_i) \geq 0$, $G'(\alpha_i) \geq 0$, and $G(\alpha_i) \leq 1$ for all $\alpha_i \in [\underline{\alpha}, \hat{\alpha}]$. Hence, $G(\alpha_i)$ satisfies all necessary conditions for a distribution function.

Finally, if

$$C'(1) - C(1) < \pi(\underline{\alpha}), \tag{A22}$$

then the equilibrium strategy includes a mass point at the acquisition rate $\hat{\alpha} = 1$, while if not, then the equilibrium strategy is continuous throughout. [ ]

Proof of Proposition 3. The proof is similar to that of Proposition 1. Let seller *i* choose acquisition rate, α_i, according to the distribution $G_i(\cdot)$, which we assume to be continuous (the following results would not change if $G_i(\cdot)$ contains discrete probability masses).

Then, by acquiring α_i customers, seller *i*'s profit is

$$\pi_i(\alpha_i) = \left\{ \int_{\alpha_{i+1}} \dots \int_{\alpha_n} [1-\alpha_{i+1}] \dots [1-\alpha_n] dG_{i+1}(\alpha_{j+1}) \dots dG_n(\alpha_n) \right\} \alpha_i p_i - C(\alpha_i). \tag{A23}$$

Referring to (A23), the revenue is linear in α_i, while the cost is convex in α_i. Hence, the profit is concave in α_i, which means that there is a unique solution. That is, seller *i* would not randomize customer acquisition. The same applies to all other sellers. [ ]

Proof of Proposition 4. Given $p_1 \geq p_2 \geq \dots \geq p_n$, from (A23), seller *i*'s profit is

$$\pi_i(\alpha_i) = \alpha_i p_i \prod_{j=i+1}^{n} [1-\alpha_j] - C(\alpha_i).^{1} \tag{A24}$$

By (A24), for seller *n*, since its price is the lowest among all sellers, its optimal acquisition is simply the solution of

[1] Technically, we need to consider the case of $p_i = p_{i+1}$ when constructing the sellers' profit. However, as we explain in the main text following Proposition 4, any tie in the (first-stage) prices cannot form an equilibrium strategy, and so for ease of exposition, we ignore the case of $p_i = p_{i+1}$ when calculating the seller's profit, (A24).

$$p_n = C'(\alpha_n). \tag{14}$$

More generally, seller $i < n$ would maximize its profit according to the first order condition

$$p_i \prod_{j=i+1}^{n} [1-\alpha_j] = C'(\alpha_i). \tag{15}$$

Substituting (14) into the corresponding first order condition, (15), for seller n-1, seller n-1's optimal acquisition is the solution of

$$p_{n-1}[1-\alpha_n] = C'(\alpha_{n-1}). \tag{A25}$$

For any $i < n$, seller i's acquisition, α_i, can be obtained by applying (15) recursively for n, n – 1, …, i. Hence, we can solve (15) recursively to obtain all equilibrium α_i. [ ]

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By (A23), at any price $p_i \in [\underline{p}, v]$, seller i's profit is

$$\begin{aligned}\pi_i(p_i) = &\int_{p_j < p_i} [p_i \alpha_i(p_i, \alpha_j(p_j)) - C(\alpha_i(p_i, \alpha_j(p_j)))] dF(p_j) \\ &+ [p_i \alpha_i(p_i) - C(\alpha_i(p_i))][1 - F(p_i)],\end{aligned} \tag{A26}$$

with $\alpha_i(p_i)$ and $\alpha_j(p_j)$ characterized by (14) and $\alpha_i(p_i, \alpha_j)$ characterized by (15).[2] Since the minimum support of seller i's pricing strategy is $\underline{p}$, at all $p_i \le \underline{p}$, $F(p_i) = F(\underline{p}) = 0$. Hence, by (A26) and (14),

$$\pi_i(p_i) = \underline{p}\alpha_i(\underline{p}) - C(\alpha_i(\underline{p})) = \underline{p}C'^{-1}(\underline{p}) - C(C'^{-1}(\underline{p})).$$

For the randomized strategy to be an equilibrium, no $p_i < \underline{p}$ should yield higher profit. Accordingly, the lower bound, $\underline{p}$, is defined by

$$\underline{p} = \arg\max \{pC'^{-1}(p) - C(C'^{-1}(p))\}. \tag{A27}$$

[2] In previous studies of price dispersion (e.g., Varian 1980; Narasimhan 1988; McAfee 1994), Lemma 1 applies. Specifically, when a seller sets price at v, its profit is determined by its own price and the deterministic acquisitions, and hence has a simple closed form. In our case, from (A26), it is clear that even if a seller sets price at v, its profit will be a function of another seller's price, and hence we must integrate over that seller's price to obtain the profit.

The preceding analysis also applies to seller *j*. Since the minimum of the support of seller *j*'s pricing strategy is also $\underline{p}$, we have $\pi_i(\underline{p}) = \pi_j(\underline{p})$, i.e., both sellers earn the same expected profit, which is the same whether the equilibrium is symmetric or asymmetric.

Generally, the heuristics of deriving the symmetric pricing strategy, $F(\cdot)$, are similar to the one that we applied in deriving the two-seller symmetric randomized-strategy equilibrium when sellers acquire consumers before pricing. In particular, we set the derivative of (A26) equal to zero, since seller *i* must earn equal profits in all prices $p_i \in [\underline{p}, v]$, and then use the boundary conditions, $F(\underline{p}) = 0$ and $F(v) = 1$, to compute $F(\cdot)$.

However, in this scenario, the profit of a seller depends on its equilibrium acquisition, and, by Lemma 2, its equilibrium acquisition is defined by an implicit function of the cost of customer acquisition. Hence, without specifying the cost function, we cannot explicitly characterize the equilibrium pricing strategy.

Constant Returns to Scale Cost Function

Suppose that customer acquisition consists of draws with replacement from a general population of L potential consumers. Let υ_m be the yield of *unique* customers, and A_m be the cumulative number of unique customers at draw *m*. By these definitions,

$$A_m = A_{m-1} + \upsilon_m, \tag{A28}$$

and the acquisition rate,

$$\alpha_m \equiv \frac{A_m}{L}. \tag{A29}$$

In draw *m*, the probability that the additional draw is a consumers who has already been drawn is A_{m-1}/L, and so, the expected yield of unique customers from draw *m* is

$$\upsilon_m = 1 - \frac{A_{m-1}}{L}. \tag{A30}$$

Substituting (A30) into (A28), the cumulative number at draw *m*

$$A_m = A_{m-1} + 1 - \left[\frac{1}{L}\right] A_{m-1} = \left[1 - \frac{1}{L}\right] A_{m-1} + 1. \tag{A31}$$

Hence,

$$A_{m-1} = \left[1 - \frac{1}{L}\right] A_{m-2} + 1.$$

Substituting into (A31),

$$A_m = \left[1 - \frac{1}{L}\right]^2 A_{m-2} + \left[1 - \frac{1}{L}\right] + 1.$$

Likewise, by recursively substituting for $A_{m-2}, A_{m-3}, ..., A_2$, and noting that the first draw yields a unique customer with certainty, i.e., $A_1 = 1$, we obtain

$$A_m = \sum_{i=1}^{m} \left[1 - \frac{1}{L}\right]^{i-1} = \sum_{i=0}^{m-1} \left[1 - \frac{1}{L}\right]^{i}. \tag{A32}$$

Summing all the *m* terms in (A32), the number of unique customers from *m* draws is

$$A = \left[1 - \left[1 - \frac{1}{L}\right]^m\right] L,$$

which implies that

$$m = \ln\left(1 - \frac{A}{L}\right) \Big/ \ln\left(1 - \frac{1}{L}\right). \tag{A33}$$

Let *c* be the unit cost of each draw. Then, using (A33) and (A29), the expected cost of acquiring *A unique* customers would be

$$C(A) = cm(A) = \frac{c \ln\left(1 - \frac{A}{L}\right)}{\ln\left(1 - \frac{1}{L}\right)} = \frac{c \ln(1 - \alpha)}{\ln\left(1 - \frac{1}{L}\right)}. \tag{A34}$$

Comparing with (16), we must have

$$\theta = -\frac{c}{\ln\left(1 - \frac{1}{L}\right)},$$

and (A34) is the only function that is consistent with our setting, where each acquisition is an independent draw of consumers with replacement. This was the setting modeled by Butters (1977), Grossman and Shapiro (1984), and McAfee (1994). [ ]