Forthcoming, *Management Science*

Buyer Uncertainty and Two-Part Pricing: Theory and Applications

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Revised, October 2009

Abstract

We consider two-part pricing of a service offered to risk-averse buyers subject to demand uncertainty. Buyers subscribe to the contract before resolution of the uncertainty. Sellers set two-part prices that trade-off between insuring buyers against the uncertainty and the ex post deadweight loss from inefficient usage.

If marginal and total benefits from the service are positively correlated (a sufficient condition is that the uncertainty not directly affects the buyer benefit), the usage charge should be set above the marginal cost of the service. If marginal and total benefits are negatively correlated, the usage charge should be set below the marginal cost. These results apply whether the seller has market power or is subject to competition.

The difference between the profit-maximizing usage charge and marginal cost increases with buyer risk aversion. Our results can be extended to the case of the seller being more risk-averse than buyers. We discuss applications to pricing of beach and ski resorts, lines of credit, utility computing, and government services.

Keywords: Pricing, Demand Uncertainty, Insurance, Risk Aversion

JEL Classifications: D81, L11, L80.

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1. Introduction

Postings on TripAdvisor.com, "Maui Forum: Bad weather":[1]

> "[P]eople should know- the weather in Maui is a crapshoot. We came here for sun and snorkeling and are disappointed. Mexico, Cuba, the Dominican – all full sun and warm."
>
> "I have been to Mexico five times and two of the times it rained. I went to Maui this February for 10 days and saw nothing but sunshine. Its [sic] hit or miss wherever you travel, not just to Maui."

Weather is a major uncertainty at beach resorts. Buyers are subject to demand uncertainty in many other settings. Other retail examples include ski resorts, health clubs, discos, lines of credit, and mobile telephony. Industrial examples include business process outsourcing and manufacturing services.[2]

Two-part prices are widely used in these settings: ski resorts offer season passes providing discounts on usage charges, health clubs charge an annual membership and plus usage fees, discos set a cover charge and prices for drinks, banks charge a lump-sum commitment fee for lines of credit plus interest on the amounts drawn, and mobile telcos offer monthly plans with charges for additional call time.

Beginning with Oi (1971), a large literature has analyzed two-part pricing. With the notable exception of Hayes (1987), previous analytical research focused on price

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[1] Posted by LeavinforMaui and MPLSCOLD, TripAdvisor.com, "Maui Forum: Bad weather", April 08, 2009, http://www.tripadvisor.com/ShowTopic-g29220-i86-k2695570-Bad_weather-Maui_Hawaii.html [Accessed, May 3, 2009].

[2] "Manufacturing services" is not an oxymoron. It is a large industry that includes multi-billion dollars giants such as Celestica, Flextronics, and Foxconn. They provide manufacturing services on contract to brand-owners of electronics and electrical goods such Hewlett-Packard and Nokia. A.T. Kearny (2004) analyzes how contract manufacturing services enables risk-sharing.

discrimination, specifically, how menus of two-part prices can induce self-selection among customer segments differing in their usage demand (Essegaier et al. 2002; Kolay and Shaffer 2003; Masuda and Whang 2006; Bagh and Bhargava 2008; Xiao et al. 2008; see especially the survey by Armstrong (2006)). However, empirical research has shown that consumers systematically choose pricing plans which are sub-optimal in the sense that other plans would have provided the same consumption at lower cost. Such sub-optimal consumer behavior has been rationalized as the outcome of ex-ante uncertainty in demand (Danaher 2002; Narayanan et al. 2007; Lambrecht et al. 2007).

Generally, risk-averse consumers who face ex-ante demand uncertainty would seek insurance. Despite the strong empirical evidence that demand uncertainty affects consumer choice, most previous research has overlooked the possible role of two-part pricing in providing buyers with insurance against demand uncertainty. Hayes (1987) showed that if some item and all other goods were complements, then a two-part pricing scheme with the usage charge below the marginal cost of the item provides insurance. However, in the various contexts mentioned – fitness clubs, bars, all-you-can-eat buffets, long-distance telephony, and video rentals – such complementarity does not seem likely.[3] The managerial applicability of Hayes' result was also limited by its implication that the optimal two-part pricing scheme (with the usage charge below the marginal cost) would reduce consumer's utility in low states and raise utility in high states. This recommendation contradicts the usual intuition that insurance works by raising utility in low states and reducing utility in high

[3] Strictly, Hayes' (1987) condition was that the covariance between the buyer's consumption of the item and her marginal utility of income be positive. Essentially, this requires that the item and all other goods be complements: in states where the buyer has a stronger preference for the item, and so consumes more of it, she must also have a stronger preference for all other goods.

states.[4]

Here, we revisit the insurance role of two-part pricing. As reviewed above, despite the limited academic literature, two-part pricing is widely applied in situations of demand uncertainty. Accordingly, it would be helpful to derive managerial guidance on the insurance role of two-part pricing. Our results are based directly on the nature of the buyers' demand uncertainty rather than complementarity in demand, yielding intuitive managerial implications in various markets. Further, our results are consistent with the intuition of providing insurance by raising income in low states and reducing income in high states. Our results apply in situations of market power as well as competition, and where buyers are risk-averse or risk-preferring.

Specifically, we analyze two-part pricing in a setting where buyers contract for some service before resolution of uncertainty. Each buyer faces a shock which shifts her marginal benefit for the service and possibly also affects her total benefit. If the marginal and total benefits are positively correlated (a sufficient condition is that the shock not directly affect total benefit), setting the usage price *above* marginal cost would reduce the quantities consumed and narrow the difference between ex post net benefits. However, if the marginal and total benefits are negatively correlated, setting the usage price *below* marginal cost would raise the quantities consumed and narrow the difference between ex post net benefits.

For risk-averse buyers, a compression of net benefits would raise ex ante expected utility. In both cases – where marginal and total benefits are positively and negatively correlated – the reduction in the difference in net benefit between the states reduces risk, and

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[4] Thakor and Udell (1987) discusssed the insurance role of "two-part pricing". However, in their setting, the buyer's usage was exogenous, and so, the "usage charge" of the two-part price was trivial.

hence, raises ex ante expected utility. However, to the extent that the usage charge differs from the marginal cost of the service, usage differs from the efficient level. The profit-maximizing usage charge balances insurance against risk with the inefficient usage.[5]

How could a Maui beach resort apply two-part pricing to mitigate risk-averse tourists' uncertainty about the weather? All tourists must pay for the hotel room, which is tantamount to the entry fee. If the weather is good, the tourist's demand for beach activities such as sailing and diving, and total benefit will be high. So, to provide insurance, the resort should price beach activities above marginal cost. However, if the weather is rainy, the tourist's demand for indoor activities such as spa treatments, gym, and aerobics will be high while her total benefit will be low. Hence, to provide insurance, the resort should price indoor activities below marginal cost.

In similar ways, two-part pricing could provide insurance in other markets: health clubs could insure consumers against weight gain, discos could insure patrons against not finding a nice match, banks could insure borrowers against liquidity shocks, and outsourced service providers could insure customers against business fluctuations.

The rest of this paper is organized as follows. In the next section, we present the profit-maximizing two-part pricing strategy in situations of uncertain demand. Section 3 shows how to adjust the two-part pricing strategy with the buyers' degree of risk aversion, and that our results are robust to the market structure. Section 4 concludes the paper with discussion of managerial implications, limitations, extensions, and future research.

2. Demand Uncertainty

[5] Our research is somewhat related to the literature on the use of reservations to insure buyers against demand uncertainty (Png 1989; Alexandrov and Lariviere 2007; Elmaghraby et al. 2007).

Buyers have initial wealth, *I*. Each buyer's demand depends on a state *s*, which is a random variable with cumulative distribution function, $G(.)$, on the interval, $[\underline{s}, \bar{s}]$. The distributions may be independent across buyers (Will I like someone in the disco?), or perfectly correlated across buyers (Will it rain in Maui next week?).

In state *s*, the buyer gets net benefit, $b(q,s)+m$, where *q* is consumption of the service and *m* is consumption of all other goods. We assume that, for all $s \in (\underline{s}, \bar{s}]$ and $q \geq 0$, the benefit satisfies

$$b_s(q,s) > 0, \quad b_q(q,s) > 0, \quad b_{qq}(q,s) < 0, \tag{1}$$

while, further, for simplicity, $b(q,\underline{s}) = 0$, and $b_q(0,s) = \bar{b}$.[6] The assumption $b_s(q,s) > 0$ is essentially a regularity condition that a higher *s* represents a better state. The condition $b_q(0,s) = \bar{b}$ implies that buyers would not buy the service if the usage charge exceeds $\bar{b}$.

We consider two cases:

- Positively correlated marginal and total benefits,

$$b_{sq}(q,s) = b_{qs}(q,s) > 0, \tag{2}$$

- Negatively correlated marginal and total benefits,

$$b_{sq}(q,s) = b_{qs}(q,s) < 0, \tag{3}$$

for all $s \geq \underline{s}$ and $q \geq 0$. By (1), $b_s > 0$, so, if $b_{qs}(q,s) > 0$, then marginal and total benefits will be positively correlated, while, if $b_{qs}(q,s) < 0$, then marginal and total benefits will be negatively correlated.[7]

To illustrate, consider the Maui tourist. If the weather is good, the tourist's demand

[6] We use prime and double prime to represent first and second derivatives of functions with single arguments, e.g., $b'(.)$ and $b''(.)$, and subscripts to represent first and second partial derivatives of functions with multiple arguments, e.g., $b_s(q,s)$ and $b_{ss}(q,s)$.

[7] The covariance of two monotone functions of a random variable is positive (Schmidt 2003).

(marginal benefit) from beach activities and total benefit would be higher than if it is rainy (positive correlation). If it is rainy, her marginal benefit from indoor activities would be higher than if the weather is good but her total benefit would be lower (negative correlation).

We assume that the seller maximizes profit and is risk-neutral. The sequence of events is as follows. First, the seller announces the two-part price, (T, p), where T is the entry fee and p is the usage charge. Then, in the second stage, buyers decide whether to subscribe to the two-part contract. If a buyer does subscribe, she pays the fee, T, following which, in the third stage, the state, s, is revealed, and she decides how much of the service to consume. If the buyer does not subscribe to the two-part contract, in the third stage, her net benefit would be simply $b(0, s) + I$, and the seller's profit would be zero.

Define a Bernoulli utility function, $u(x)$, with $u'(x) > 0$, over the (ex post) realizations of the buyer's net benefit,

$$x(T, p, s) \equiv b(q(T, p, s), s) + m(T, p, s), \qquad (4)$$

where q is consumption of the service, and m is consumption of all other goods. We assume that buyers are risk-averse, $u''(x) < 0$.

The seller will set the usage charge $p < \bar{b}$, otherwise no buyer would buy. Consider a buyer who has subscribed to the contract. In the third stage, after resolution of the uncertainty, s, she chooses q and m to maximize net benefit,

$$b(q, s) + m, \qquad (5)$$

subject to the budget constraint,

$$T + pq + m \leq I. \qquad (6)$$

Hence, consumption in state s, denoted $(q(T, p, s), m(T, p, s))$, is characterized by the first-order condition,

$$b_q(q(T,p,s),s) = p\,, \tag{7}$$

and the binding budget constraint,

$$m(T,p,s) = I - T - pq(T,p,s)\,. \tag{8}$$

Note that, conditional on the buyer subscribing, her usage of the service, $q(T,p,s)$, does not depend on the entry fee, *T*, i.e.,

$$q_T(T,p,s) = 0\,. \tag{9}$$

Substituting from (7) and (8) in (4), in state *s*, the buyer's (ex post) net benefit is

$$x(T,p,s) \equiv b(q(T,p,s),s) + m(T,p,s) = b(q(T,p,s),s) + I - T - pq(T,p,s). \tag{10}$$

Differentiating, substituting from (7) and using (1),

$$\frac{\partial x}{\partial s} = \frac{\partial b}{\partial q}\frac{\partial q}{\partial s} + \frac{\partial b}{\partial s} - p\frac{\partial q}{\partial s} = \left[\frac{\partial b}{\partial q} - p\right]\frac{\partial q}{\partial s} + \frac{\partial b}{\partial s} = \frac{\partial b}{\partial s} > 0\,,$$

which proves the following result.

Lemma 1: *The buyer's ex post net benefit is increasing with respect to s, i.e.,* $x_s > 0$.

In the first stage, the seller chooses the entry fee, *T*, and usage charge, *p*, to maximize profit

$$\pi = T + \int_{\underline{s}}^{\bar{s}} [p-c]q(T,p,s)dG(s)\,, \tag{11}$$

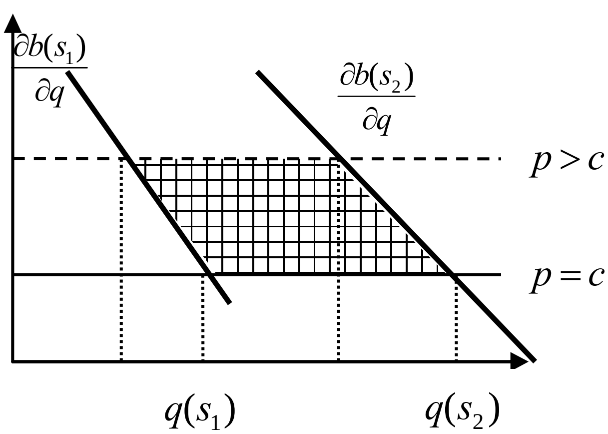
subject to the buyer's maximization of net benefit, (7) and (8), and the buyer preferring to subscribe to the two-part pricing contract (individual rationality),

$$\int_{\underline{s}}^{\bar{s}} u(x(T,p,s))dG(s) \geq \int_{\underline{s}}^{\bar{s}} u(b(0,s)+I)dG(s) \equiv \bar{u}\,. \tag{12}$$

To maximize profit, the seller sets the margin, $p-c$, to balance between two factors. One is providing the buyer with ex ante insurance against demand uncertainty, while the other is the ex post deadweight loss from inefficient consumption to the extent that the usage charge deviates from the marginal cost. The seller then sets the entry fee, *T*, to mop up the buyer surplus. Our main result shows that the profit-maximizing scheme involves a positive margin if marginal and total benefits are positively correlated, and a negative margin if marginal and total benefits are negatively correlated.[8]

Proposition 1: *The profit-maximizing contract entails a usage charge greater than the marginal cost,* $p > c$*, if marginal and total benefits are positively correlated, and a usage charge less than the marginal cost,* $p < c$ *if marginal and total benefits are negatively correlated. In both cases, the entry fee,* $T \geq 0$*, and is decreasing in the usage charge, p.*

Figure 1. Positively correlated marginal and total benefits



To appreciate the intuition of positively correlated marginal and total benefits, suppose that the seller sets the usage charge at the marginal cost, $p = c$, and the entry fee such that the buyer is just indifferent about subscribing to the contract. Figure 1 shows the buyer's demand for the service in states, $s_1 < s_2$, where s_1 and s_2 represent bad and good

[8] For brevity, the proofs of this and subsequent results are presented in the online Appendix A.

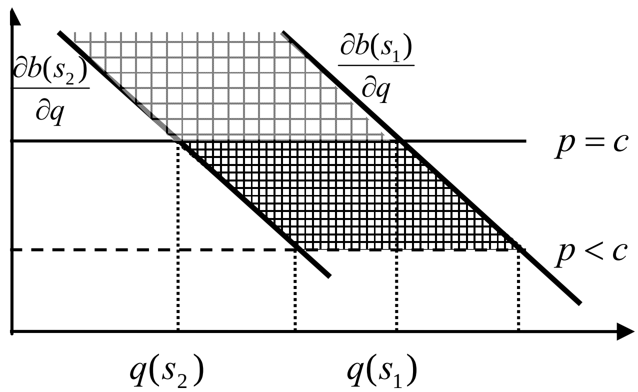
weather respectively.

Given usage charge, $p = c$, the buyer maximizes net benefit in accordance with (7), choosing quantities, $q(s_1) < q(s_2)$. The buyer's total benefit from the service less usage payment in state s_1 is the triangle under the marginal benefit curve down to the price line. Likewise, the buyer's total benefit from the service less usage payment in state s_2 is the triangle under the marginal benefit curve down to the price line. The difference in these total benefits less usage payments equals the difference in the buyer's net benefits between the states. This difference is the essential source of the buyer's risk. In Figure 1, it is represented by the area between the two marginal benefit curves down to the price line.

Now, suppose that the seller raises the usage charge to $p > c$. The buyer would reduce consumption of the service in both states. Referring to Figure 1, the difference between the buyer's net benefits in the two states would shrink – by the shaded area. Hence, by raising the usage charge, the seller reduces the difference between the buyer's net benefits in the two states, and hence reduces her risk.

The seller can reduce the entry fee while still ensuring that the buyer would subscribe to the contract. However, the increase in usage charge would reduce consumption from the efficient level. On balance, the seller's profit increases as, by providing insurance, the seller gains more through the higher usage charge than loses from the lower entry fee.

Figure 2. Negatively correlated marginal and total benefits



To understand the intuition of negatively correlated marginal and total benefits, suppose that the seller sets the usage charge at the marginal cost, $p = c$, and the entry fee such that the buyer is just indifferent about subscribing to the contract. Figure 2 shows the buyer's demand for the service in bad (s_1) and good weather (s_2) respectively.

Given usage charge, $p = c$, the buyer maximizes net benefit in accordance with (7), choosing usage, $q(s_2) < q(s_1)$. Compare the buyer's net benefit in bad vis-a-vis good weather. The difference in the buyer's net benefits (total benefit less usage payments) between the states is

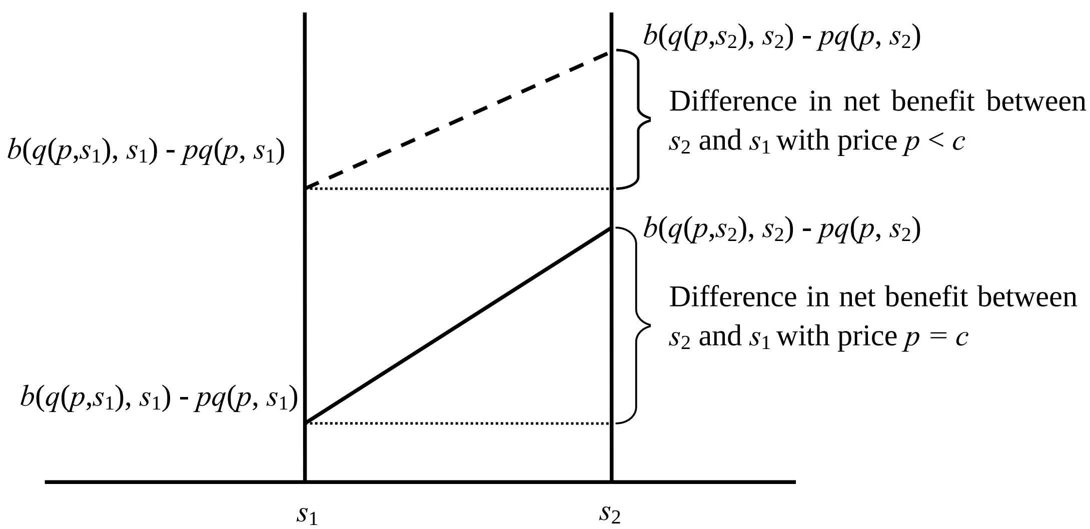
$$[b(q(T,p,s_2),s_2) - pq(T,p,s_2)] - [b(q(T,p,s_1),s_1) - pq(T,p,s_1)] > 0. \qquad (13)$$

In Figure 2, it is represented by the *negative* of the solid yellow area between the two marginal benefit curves down to the price line, in addition to a lump-sum difference, $b(0,s_2) - b(0,s_1) > 0$, not illustrated in the figure. By Lemma 1, $x(s_2) > x(s_1)$, hence the difference in the shocks to net benefit (without any usage), $b(0,s_2) - b(0,s_1)$, dominates the solid yellow area.

Now, suppose that the seller *reduces* the usage charge to $p < c$. The buyer would increase usage in both states. Referring to Figure 2, the solid yellow area would expand by the hatched yellow area. Since the yellow area appears negatively in the difference in net benefit, (13), the expansion of the yellow area would *reduce* the difference in net benefit. Accordingly, by reducing the usage charge, the seller would reduce the difference between the buyer's net benefit in the two states, and hence reduce the risk.

Figure 3 depicts the intuition of negatively correlated marginal and total benefits from another perspective. If the seller reduces the usage charge below the marginal cost, the buyer will be induced to increase usage in both states. The key is that, as $b_{qs}(q,s) < 0$, the increase in usage would be relatively larger in bad weather (s_1) than in good weather (s_2). Accordingly, this would reduce the difference in the net benefit between bad and good weather, and so, provide insurance.

Figure 3. Negatively correlated marginal and total benefits, another view



For risk-averse buyers, a compression of net benefits would raise expected utility. In both cases – where marginal and total benefits are positively and negatively correlated – the reduction in the difference in net benefit between the states reduces risk, and hence, raises ex ante expected utility. The two-part pricing scheme provides insurance by raising net benefit in the low states and reducing net benefit in the high state. However, to the extent that the usage charge differs from the marginal cost of the service, the buyer would consume an inefficient quantity of the service. The profit-maximizing usage charge balances insurance against risk with the inefficient usage.

The first part of Proposition 1 characterizes the relation of the profit-maximizing usage charge to the marginal cost of the service. An immediate issue is the entry fee and its relation, if any, to the usage charge. The second part of the Proposition shows that the entry fee should be positive and that a higher usage charge is associated with lower entry fee, and vice versa. These results are driven by the buyer's individual rationality constraint. The seller should raise the entry fee up to the point that the individual rationality constraint binds. If the seller raises the usage charge, then, to ensure that the buyer would still subscribe to the two-part price, the seller must reduce the entry fee.

Proposition 1 shows clearly that the insurance role of two-part pricing is much more general than suggested by Hayes (1987). To place Hayes' contribution and ours in context, consider an overarching framework in which buyers have Bernoulli utility function, $u(x)$, over ex post realizations of the net benefit, $x(q, m, s)$, from consumption, q, of the service, consumption of all other goods ("income"), m, and the state, s. Hayes implicitly assumed linear utility, $u(x) = x$, with a general ex post state-dependent net benefit, $x(q, m, s)$.[9] Then, buyers prefer a two-part price with the usage charge below the marginal cost of the service if and only if the covariance between consumption of the service, $q(s)$, and the "marginal utility of income", $x_m(s)$, is positive or equivalently, the consumption and all other goods (income)

[9] Hayes (1987) termed the net benefit "utility" as she implicitly ignored the Bernoulli utility.

are complements.

By contrast, within the same overarching framework, we allow general utility, $u(x)$, with additively separable ex post net benefit, $x(q, m, s) = b(q, s) + m$. In our setting, the uncertainty can clearly be identified as buyer's uncertainty about demand (marginal benefit) for the service. While Hayes' (1987) result turns on the covariance of the buyer's consumption of the service with her "marginal utility of income", we require only one condition – that the uncertainty systematically affects the buyer's demand (marginal benefit) for the service.[10]

In particular, it is not necessary for the uncertainty to have any direct effect on total benefit. To be specific, suppose that the uncertainty has no direct effect on total benefit, i.e., $b(0,s) = 0$. Now

$$b_s(q,s) = \frac{\partial}{\partial s} b(q,s) = \frac{\partial}{\partial s}\int_0^q b_q(q,s)dq = \int_0^q b_{qs}(q,s)dq\,, \tag{14}$$

hence, $b_{qs} > 0$ implies that $b_s > 0$. Thus, even with $b(0,s) = 0$, the condition $b_{qs} > 0$ alone is sufficient to imply that $b_s > 0$, and so, by Lemma 1, $x_s > 0$, and thus, by Proposition 1, that the usage charge should exceed the marginal cost, i.e., $p > c$. Hence, even if the uncertainty has some negative direct effect on total benefit, $b(0,s) < 0$, but not too large, the profit-maximizing strategy usage charge will exceed marginal cost, $p > c$. We note that, in the case of $b_{qs} < 0$, it is necessary that the uncertainty have sufficiently large negative direct effect on total benefit, $b(0,s) < 0$, for the profit-maximizing usage charge to fall short of the marginal cost, $p < c$.

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[10] Within the same framework, the model of Thakor and Udell (1987) can be characterized as allowing general utility but with the very specific ex post net benefit, $x = s + m$. In this set-up, the buyer's usage is exogenous, and so, the pricing scheme includes a "usage charge" only in a trivial way, and the pricing scheme was comprised "two parts" only in a trivial sense.

Our analysis can be readily extended to situations where the buyer is risk-neutral or risk-preferring. The following result is a straightforward implication of Proposition 1 with $u''(x) = 0$, and relates our analysis to the classic contribution of Oi (1971).

Corollary 1 (Oi 1971): *If buyers are risk-neutral, then the profit-maximizing contract involves a usage charge equal to the marginal cost,* $p = c$.

Moreover, we can also show that if buyers are risk-preferring, i.e., $u''(x) > 0$, then the profit-maximizing contract involves a usage charge less than the marginal cost, $p < c$, if marginal and total benefits are positively correlated, and $p > c$ if marginal and total benefits are negatively correlated. Essentially, risk-preferring buyers would benefit from *stretching* the disparity in net benefit between the low and high states. Referring to Figure 1, the stretch can be achieved by reducing the usage charge below the marginal cost of the service, while, referring to Figure 2, the stretch can be achieved by raising the usage charge above marginal cost.[11]

3. Risk Aversion and Competition

We have characterized the profit-maximizing two-part price to buyers who face demand uncertainty. Over time and across geographical and product markets, buyers may differ in the degree of risk aversion. Hence, an important managerial issue is how the seller should adjust the two-part price, and specifically, the usage charge, according to differences in buyer

[11] These arguments make clear that, for purposes of insuring the buyer against risk, whether the usage rate should be higher or lower than the seller's marginal cost depends on the shape of the (ex ante) utility function, but not the structure of the (ex post) net benefit. By contrast, Hayes (1987) based her results on the structure of the net benefit, specifically, a positive covariance between consumption of the service and the marginal benefit of income.

risk aversion.

With risk-averse buyers, Proposition 1 showed that the seller should adjust the usage charge away from marginal cost, so as to reduce the disparity in net benefit across the various states, and provide insurance to buyers. It would seem intuitive that the difference between the usage charge and marginal cost would increase in the buyers' degree of risk aversion. To address this conjecture, we need to parameterize buyer risk aversion.

Following research in financial economics, we assume that buyers exhibit constant absolute risk aversion as represented by the exponential Bernoulli utility function,

$$u(x) = -e^{-\alpha x}, \tag{15}$$

where α is the coefficient of absolute risk aversion (Pratt 1964; Arrow 1965). The following result is the basis for proving that the difference between the usage charge and marginal cost increases with buyer risk aversion.

Lemma 2: *Suppose that buyer utility is represented by* $u(x) = -e^{-\alpha x}$. *Then the seller's profit function is supermodular in p and α, i.e.,* $\partial^2 \pi / \partial p \partial \alpha \geq 0$.[12]

From Lemma 2, we immediately have Proposition 2, which confirms the simple intuition that, since the seller raises the usage charge above marginal cost in order to provide insurance, if buyers are more risk averse, the seller should raise the usage charge even more, and concomitantly reduce the entry fee. Similarly, in the case of negatively correlated marginal and total benefits, if buyers are more risk averse, the seller should further reduce the usage charge below marginal cost and concomitantly raise the entry fee.

[12] Please refer to Amir (2005) for a simple exposition of supermodularity and its application to economic analysis.

Proposition 2: *Suppose that buyer utility is represented by* $u(x) = -e^{-\alpha x}$. *If marginal and total benefits are positively correlated, the profit-maximizing usage charge, p, is non-decreasing and the entry fee, T, is non-increasing in the buyers' degree of risk aversion, α. If marginal and total benefits are negatively correlated, the usage charge, p, is non-increasing and the entry fee, T, is non-decreasing in the buyers' risk aversion, α.*

As Hayes (1987) noted, two-part pricing is widespread in competitive industries. While Hayes' analysis was limited to the competitive context, our main result applies in settings of market power as well as competition. Proposition 1 proved the result in the context of market power. Here, we prove the robustness of the result by showing that it also applies in a competitive market subject to free entry.

Specifically, let the situation be as defined as in section 2, except that, in the first stage, multiple sellers compete to provide buyers with the identical service. Suppose that each seller chooses the entry fee, *T*, and usage charge, *p*, to maximize buyers' ex-ante utility

$$\int_{\underline{s}}^{\bar{s}} u\big(b(q(T,p,s)+m(T,p,s)\big)dG(s) \qquad (16)$$

subject to non-negative profit,

$$T + \int_{\underline{s}}^{\bar{s}} [p-c]q(T,p,s)dG(s) \geq 0\,. \qquad (17)$$

where the buyer's usage of the service, $q(T,p,s)$, and consumption of all other goods, $m(T,p,s)$, are given by (7) and (8) respectively. By contrast with the situation of market power, the competitive pricing strategy is driven by each individual seller's zero profit constraint rather than the buyer's individual rationality constraint.

The two-part contract defined by (16) and (17) is an equilibrium. The reasoning is as follows. By construction, the contract maximizes buyers' ex-ante utility subject to the seller earning non-negative profit. Hence, there is no way for another seller to offer a contract that provides buyers with higher ex-ante utility and still make non-negative profit.

Proposition 3: *With free-entry competition, the equilibrium two-part pricing strategy involves a usage charge greater than the marginal cost,* $p > c$, *with negative entry fee,* $T < 0$, *if marginal and total benefits are positively correlated, and a usage charge less than the marginal cost,* $p < c$, *with positive entry fee,* $T > 0$, *if marginal and total benefits are positively correlated*.

If the buyer's marginal and total benefits are positively correlated, the equilibrium two-part pricing strategy entails a usage charge above marginal cost. Hence, in the third stage, each seller would make a positive profit contribution. In the first stage, with free-entry competition, each seller must earn zero expected profit. Accordingly, the entry fee must be negative, $T < 0$ (a "sign-up bonus"). Competing sellers might cream-skim by offering lower prices that still cover marginal cost but without contract. Realistically, however, providing the service may involve some fixed overhead, such as for account maintenance. In competitive equilibrium, the entry fee must be net of these per-customer fixed costs. To the extent of such per-customer fixed costs, other sellers would be deterred from cream-skimming.

If the services offered by the competing sellers are differentiated, the competitive equilibrium service plans depend on the nature of differentiation. Nevertheless, our main result should continue to hold, since each seller can be viewed as a monopolist given other sellers' pricing and product strategies. The degree to which the usage charges will differ from marginal costs will depend on the exact basis of competition and nature of service differentiation.

4. Concluding Remarks

We have shown that two-part pricing can provide risk-averse buyers with insurance against demand uncertainty under very general conditions on the buyer's utility function and the market structure. Generally, the difference between the profit-maximizing usage charge and the marginal cost of the service trades off the gain in buyers' ex ante expected utility from insurance against the ex post inefficiency in usage. Further, we showed that the extent to which the usage charge should deviate from the marginal cost increases with the buyers' degree of risk aversion. Accordingly, our analysis provides robust managerial guidance on applying two-part pricing to provide buyers with insurance against demand uncertainty.[13] [14]

Our analysis can be directly applied to various consumer contexts – beach and ski resorts with uncertain weather, health clubs with patrons uncertain about weight gain, and banks with borrowers uncertain about their finances. Typically, banks set two-part prices for lines of credit, comprising a non-refundable commitment fee and a specified spread over a

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[13] In principle, the provision of service and insurance could be separated, for instance, a business could buy IT services from one provider and insurance against demand uncertainty from another provider. However, practically, an integrated provider of both service and insurance can reduce administrative costs and also mitigate possible moral hazard between service and insurance providers. For instance, airlines and hotels allow customers to cancel or change reservations with limited or no penalties, which effectively provides customers with insurance against changes in their personal circumstances (Png 1989).

[14] Another issue of managerial importance is how to adjust the pricing strategy in response to shifts in the distribution of demand (marginal benefit). Intuitively, it might seem that: (i) if the marginal benefit is higher in the sense of first-order stochastic dominance, then the usage charge should be higher; and (ii) if the uncertainty is larger, then the difference between the usage charge and marginal cost should be larger, since buyers would want more insurance. In the online Appendix B, we present a counter-example to show that intuition (i) is not correct and prove intuition (ii). However, the managerial significance of finding (ii) is somewhat limited as it relies on a condition that might be difficult to interpret in practice.

benchmark interest rate on the amount drawn. The commitment fee is a lump sum entry fee as the borrower must pay the fee regardless of how much he draws on the facility. If the borrower draws on the facility, then he must pay the benchmark interest rate plus the specified spread on the amount drawn. Lines of credit provide borrowers with insurance (Thakor and Udell 1987; Saidenberg and Strahan 1999). Our results provide guidance on pricing of lines of credit: banks should set lower credit spreads coupled with higher commitment fees to more risk-averse borrowers.

Our analysis can also be applied to utility computing and other industrial contexts. With on-demand utility computing, the customer pays for services on demand rather than buying entire systems and software. Utility computing provides "a risk management instrument for the customer, similar to insurance" (Paleologo 2004: 20-21). Rather than set cost-plus prices for such services (Paleologo 2004), vendors could follow our analysis and apply two-part pricing to insure buyers against demand uncertainty. Vendors should pay particular attention to the direction of the uncertainty, setting usage charges above marginal cost for services like data storage for which marginal and total benefits are positively correlated, and setting usage charges below marginal cost for services like business continuity for which marginal and total benefits are negatively correlated.

Our analysis can also be applied to public policy. Governments provide re-training and job placement services to their citizens, which are financed through taxation. Citizens face uncertainty about employment. If they are unemployed, their marginal benefit from re-training and job placement would be higher than if they have a job. By our analysis, governments would maximize welfare by pricing re-training and job placement services below cost and so provide insurance against unemployment. The government could adjust taxes accordingly. Religious institutions provide wedding as well as marriage counseling services, which are financed through contributions by believers. If single church/temple members find a potential spouse, their marginal benefit from wedding services would be higher. So, to maximize welfare, religious institutions should charge above marginal cost for wedding services. However, if married church/temple members should encounter severe disagreements, their total benefit would be lower but marginal benefit from marriage counseling would be higher. So, religious institutions should charge below marginal cost for marriage counseling.

Our main analysis was set in the context of a risk-neutral seller facing risk-averse buyers. If the seller is not risk-neutral, the pricing strategy must account for the seller's risk aversion. Intuitively, the difference between the profit-maximizing usage charge and the marginal cost depends on the seller's relative to the buyers' degree of risk aversion. Subject to this adjustment, our analysis continues to apply. In particular, our analytical results in the context of risk-preferring buyers would apply to the scenario where the seller is more risk averse than buyers.

Our work suggests three directions for future work. Formally, we assumed that the buyers' demands have the identical distribution function.[15] To the extent that buyer utility and seller profits are continuous, our results would continue to apply if the buyers' demands deviated slightly from this assumption. Nevertheless, one direction for future research is to

[15] If there are n buyers with independent and identical distributions, $G_i(s)$, then the problem would be to maximize the seller's profit from n buyers subject to n buyer maximization and individual rationality constraints. Since the $G_i(s)$ are i.i.d, the problem simplifies to the problem with one buyer scaled up by n.

investigate two-part pricing when buyers' demands have different distributions If this analysis takes account of self-selection among buyers with different demands as well as insurance, it would necessarily be quite complex.

The second direction is accounting for possible psychological biases in consumer responses to two-part pricing (Gourville and Soman 2002; DellaVigna and Malmendier 2004 and 2006; Ellison 2006; Lambrecht and Skiera 2006). It is important to analyze how to adjust the two-part pricing strategy accordingly.

The third direction is to investigate the use of two-part pricing in the context of uncertainty in the distribution channel. It is well known that two-part pricing resolves the double marginalization problem in distribution (Moorthy 1987). Recent research into distribution has focused on various departures from the simple dyadic channel, such as retailer dominance and fairness (Raju and Zhang 2005; Cui, Raju, and Zhang 2007). It would be important to investigate the insurance role of two-part pricing in the distribution channel.

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