CS3245

Information Retrieval

7

Lecture 7: Scoring, Term Weighting and the Vector Space Model

Last Time: Index Compression

- Collection and vocabulary statistics: Heaps' and Zipf's laws
- Dictionary compression for Boolean indexes
  - Dictionary string, blocks, front coding
- Postings compression: Gap encoding

collection (text, xml markup etc)	3,600.0	MB
collection (text)	960.0	
Term-doc incidence matrix	40,000.0	
postings, uncompressed (32-bit words)	400.0	
postings, uncompressed (20 bits)	250.0	
postings, variable byte encoded	116.0	

Today: Ranked Retrieval

Skipping over Section 6.1; will return to it later

- Scoring documents
- Term frequency
- Collection statistics
- Weighting schemes
- Vector space scoring

Ranked retrieval

- Thus far, our queries have all been Boolean.
  - Documents either match or don't.
- **Good** for expert users with precise understanding of their needs and the collection.
  - Also good for applications: Applications can easily consume 1000s of results.
- **Not good** for the majority of users.
  - Most users incapable of writing Boolean queries (or they are capable, but they think it's too much work).
  - Most users don't want to wade through 1000s of results.
    - This is particularly true of web search.

Problem with Boolean search: Feast or Famine

- Boolean queries often result in either too few (=0) or too many (1000s) results.
- Query 1: "*standard user dlink 650*" → 200,000 hits
  - Also called "information overload"
- Query 2: "*standard user dlink 650 no card found*" → 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
  - AND gives too few; OR gives too many

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in **ranked retrieval models**, the system returns an ordering over the (top) documents in the collection with respect to a query
- **Free text queries**: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- Two separate choices, but in practice, ranked retrieval models are associated with free text queries

Ranked retrieval

- When a system produces a ranked result set, large result sets are not an issue
  - We just show the top *k* (≈ 10) results
  - We don't overwhelm the user

  - Premise: the ranking algorithm works

Scoring as the basis of ranked retrieval

- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank the documents in the collection with respect to a query?

- Assign a score – say in [0, 1] – to each document
- This score measures how well document and query "match".

Query-document matching scores

- We need a way of assigning a score to a query/document pair

- Let's start with a one-term query
- If the query term does not occur in the document: score should be 0
- The more frequent the query term in the document, the higher the score (should be)

Take 1: Jaccard coefficient

- Recall the Jaccard coefficient from Chapter 3 (spelling correction): A measure of overlap of two sets *A* and *B*

Jaccard $(A,B) = |A \cap B| / |A \cup B|$

Jaccard $(A,A) = 1$

Jaccard $(A,B) = 0$ if $A \cap B = 0$

Pros:

- *A* and *B* don't have to be the same size.
- Always assigns a number between 0 and 1.

Blanks on slides, you may want to fill in

Jaccard coefficient: Scoring example

- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?
- Query: *ides of march*
- Document 1: *caesar died in march*
- Document 2: *the long march*

Issues with Jaccard for scoring

1. It doesn't consider ***term frequency*** (how many times a term occurs in a document)
   - Rare terms in a collection are more informative than frequent terms. Jaccard doesn't consider this information
2. We need a more sophisticated way of normalizing for length
   - Later in this lecture, we'll use $|A \cap B|/\sqrt{|A \cup B|}$ . . . instead of $|A \cap B|/|A \cup B|$ (Jaccard) for length normalization.

Recap: Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a **binary vector** $\in \{0,1\}^{|V|}$

Term-document count matrices

- Store the number of occurrences of a term in a document:
  - Each document is a **count vector** in $\mathbb{N}^v$: a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Bag of words model

- Con: Vector representation doesn't consider the ordering of words in a document
  - *John is quicker than Mary* and *Mary is quicker than John* have the same vectors

- In a sense, this is a step back: The positional index was able to distinguish these two documents.
- We will look at "recovering" positional information later in this course.
- For now: bag of words model

Term frequency tf

- The term frequency $\mathrm{tf}_{t,d}$ of term *t* in document *d* is defined as the number of times that *t* occurs in *d*.
- We want to use tf when computing query-document match scores. But how?
- Raw term frequency is not what we want:
  - Relevance does not increase proportionally with term frequency.
  - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence. But not 10 times more relevant.

Note: frequency = count in IR

Log-frequency weighting

- The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

e.g. 0 → 0, 1 → 1, 2 → 1.3, 10 → 2, 1000 → 4, etc.

- Score for a document-query pair: sum over terms *t* in both *q* and *d*:

$$\text{score} = \sum_{t \in q \cap d} (1 + \log \mathrm{tf}_{t,d})$$

- The score is 0 if none of the query terms is present in the document.

Document frequency

- Rare terms are more informative than frequent terms
  - Recall stop words
  - Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
- A document containing this term is very likely to be relevant to the query *arachnocentric*
  - We want a high weight for rare terms like *arachnocentric*.

Blanks on slides, you may want to fill in

Document frequency, continued

- **Frequent terms** are less informative than **rare terms**
- Consider a query term that is frequent in the collection (e.g., *high, increase, line*)
- A document containing such a term is more likely to be relevant than a document that doesn't …
- For frequent terms, we want high positive weights for words like *high, increase,* and *line* …
- We will use document frequency (df) to capture this.

idf weight

- df_t is the <u>document</u> frequency of t: the number of documents that contain t
  - df_t is an inverse measure of the informativeness of t
  - $\mathrm{df}_t \leq N$
- We define the idf (inverse document frequency) of t by

$$\mathrm{idf}_t = \log_{10} (N/\mathrm{df}_t)$$

  - We use log (N/df_t) instead of N/df_t to "dampen" the effect of idf.

Example: suppose N = 1 million

term	df_t	idf_t
calpurnia	1	6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

$$\mathrm{idf}_t = \log_{10} (N/\mathrm{df}_t)$$

There is one idf value for each term t in a collection.

Blanks on slides, you may want to fill in

Effect of idf on ranking

- Does idf have an effect on ranking for one-term queries, like iPhone?
- idf has on ranking one term queries
  - idf affects the ranking of documents for queries with at least
  - For the query **capricious person**, idf weighting makes occurrences of **capricious** count for much more in the final document ranking than occurrences of **person**.

Collection vs. Document frequency

- The collection frequency of *t* is the number of occurrences of *t* in the collection, counting multiple occurrences.
- Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

- Which word is a better search term (and should get a higher weight)?

tf-idf weighting

- The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$\mathrm{w}_{t,d} = (1 + \log \mathrm{tf}_{t,d}) \times \log_{10}(N / \mathrm{df}_t)$$

- **Best known weighting scheme IR**
  - Note: the "-" in tf-idf is a hyphen, not a minus sign!
  - Alternative names: tf.idf, tf x idf
- **Increases** with the number of occurrences within a document
- **Increases** with the rarity of the term in the collection

Final ranking of documents for a query

$$\text{Score}(q,d) = \sum_{t \in q \cap d} \text{tf.idf}_{t,d}$$

Binary → count → weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

Documents as vectors

- So we have a |V|-dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- High-dimensional: tens of thousands of dimensions; each dictionary term is a dimension
- These are very sparse vectors - most entries are zero.

Queries as vectors

- Key idea 1: Do the same for queries: represent them as vectors in the space; they are "mini-documents"
- Key idea 2: Rank documents according to their proximity to the query in this space

- proximity = similarity of vectors
- proximity ≈ inverse of distance
- Motivation: Want to get away from the you're-either-in-or-out Boolean model.
- Instead: rank more relevant documents higher than less relevant documents

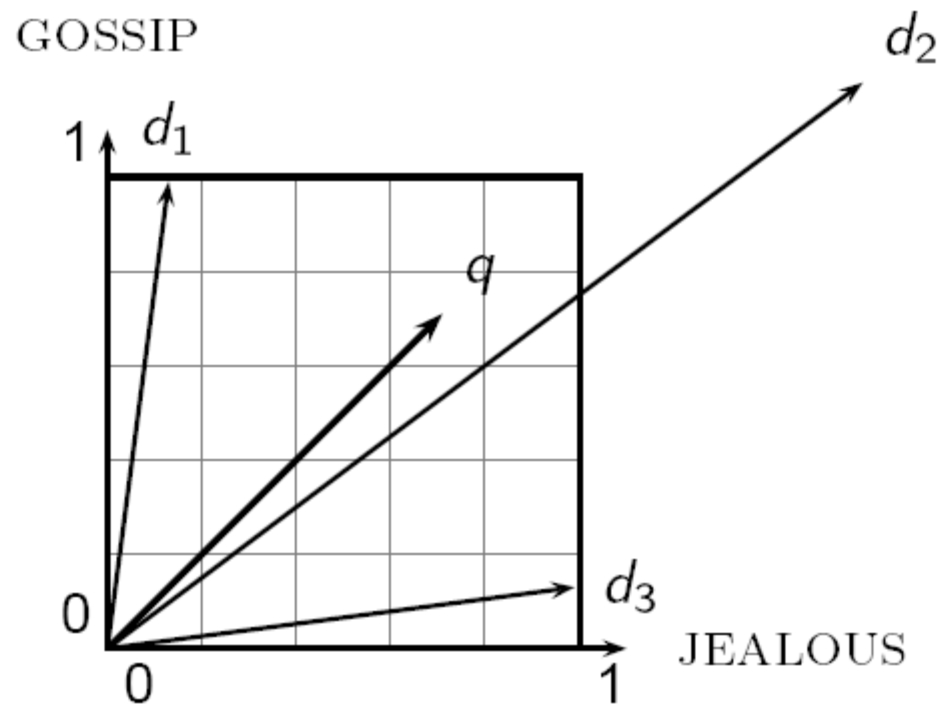
Blanks on slides, you may want to fill in

Formalizing vector space proximity

- First cut: distance between two points
  - (= distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea …

Why distance is a bad idea

The Euclidean distance between $\vec{q}$ and $\vec{d_2}$ is large even though the distribution of terms in the query $\vec{q}$ and the distribution of terms in the document $\vec{d_2}$ are very similar.



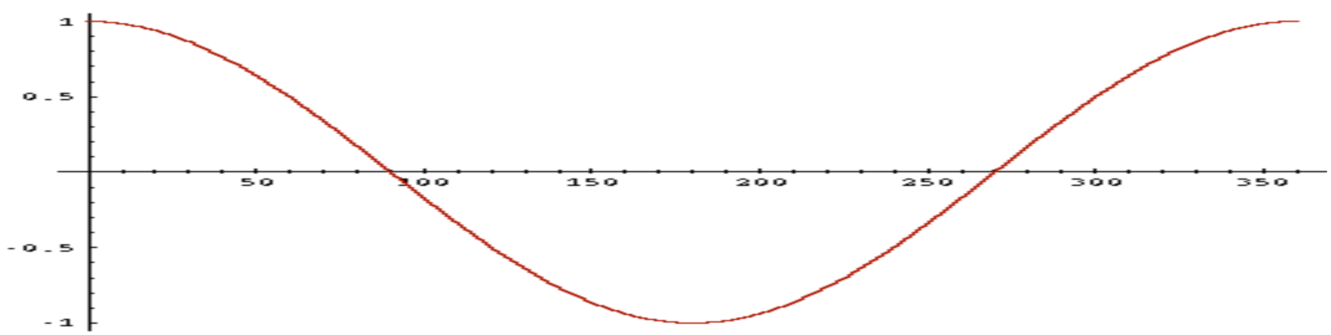
Use angle instead of distance

- Distance counterexample: take a document *d* and append it to itself. Call this document *d*′.
- "Semantically" d and d' have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity.

- Key idea: Rank documents according to angle with query.

From angles to cosines

- The following two notions are equivalent.
  - Rank documents in <u>decreasing</u> order of the angle between query and document
  - Rank documents in <u>increasing</u> order of cosine(query,document)
- Cosine is a monotonically decreasing function for the interval [0^o, 180^o]



Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L_2 norm:

$$\left\| \vec{x} \right\|_2 = \sqrt{\sum_i x_i^2}$$

- Dividing a vector by its L_2 norm makes it a unit (length) vector (on surface of unit hypersphere)
- Effect on the two documents d and d' (d appended to itself) from earlier slide: they have identical vectors after length normalization.
  - Long and short documents now have comparable weights

cosine (query, document)

Dot product

Unit vectors

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q}\bullet\vec{d}}{|\vec{q}||\vec{d}|} = \frac{\vec{q}}{|\vec{q}|}\bullet\frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the tf-idf weight of term *i* in the query
d_i is the tf-idf weight of term *i* in the document

$\cos(\vec{q},\vec{d})$ is the cosine similarity of $\vec{q}$ and $\vec{d}$ … or, equivalently, the cosine of the angle between $\vec{q}$ and $\vec{d}$.

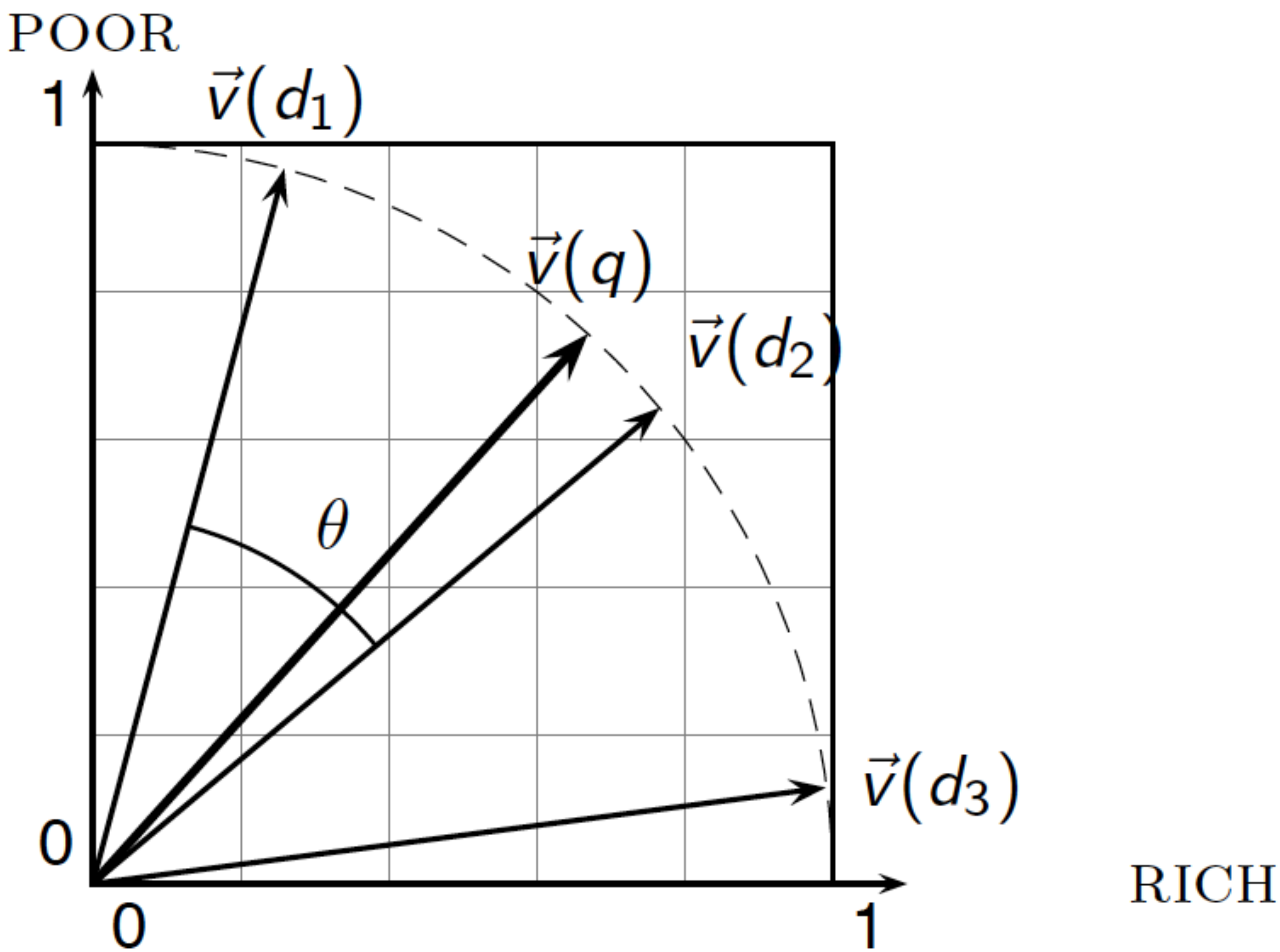
Cosine for length-normalized vectors

- For length-normalized vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q},\vec{d}) = \vec{q} \bullet \vec{d} = \sum_{i=1}^{|V|} q_i d_i$$

for q, d length-normalized.

Cosine similarity illustrated



Cosine similarity among 3 documents

How similar are the novels

SaS: *Sense and Sensibility*

PaP: *Pride and Prejudice*, and

WH: *Wuthering Heights*?

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we don't do idf weighting.

3 document example (cont'd)

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

After length normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

cos(SaS,PaP) ≈

$0.789 \times 0.832 + 0.515 \times 0.555 + 0.335 \times 0.0 + 0.0 \times 0.0$

≈ 0.94

cos(SaS,WH) ≈ 0.79

cos(PaP,WH) ≈ 0.69

Computing cosine scores


```
COSINESCORE(q)
float Scores[N] = 0
float Length[N]
for each query term t
do calculate w_{t,q} and fetch postings list for t
    for each pair(d, tf_{t,d}) in postings list
    do Scores[d] += w_{t,d} × w_{t,q}
Read the array Length
for each d
do Scores[d] = Scores[d]/Length[d]
return Top K components of Scores[]
```

tf-idf weighting has many variants

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \ldots + w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times \mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N - \mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/CharLength^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1 + \log(\mathrm{tf}_{t,d})}{1 + \log(\mathrm{ave}_{t \in d}(\mathrm{tf}_{t,d}))}$				

Columns headed 'n' are acronyms for weight schemes.

Quick Question: Why is the base of the log in idf immaterial?

Weighting may differ in queries vs documents

- Many search engines allow for different weightings for queries vs. documents
- SMART Notation: denote combination used with the notation *ddd.qqq,* using acronyms from previous table
- A very standard weighting scheme is **lnc.ltc**
- Document: logarithmic tf (**l** as first character), **n**o idf and **c**osine normalization

A bad idea?

- Query: logarithmic tf (**l** in leftmost column), idf (**t** in second column), and **c**osine normalization

tf-idf example: lnc.ltc

Document: *car insurance auto insurance*
Query: *best car insurance*

Term	Query						Document				Prod
	tf-raw	tf-wt	df	idf	wt	n'lize	tf-raw	tf-wt	wt	n'lize	
auto	0	0	5000	2.3	0	0	1	1	1	0.52	0
best	1	1	50000	1.3	1.3	0.34	0	0	0	0	0
car	1	1	10000	2.0	2.0	0.52	1	1	1	0.52	0.27
insurance	1	1	1000	3.0	3.0	0.78	2	1.3	1.3	0.68	0.53

Quick Question: what is *N*, the number of docs?

Doc length $=\sqrt{1^2+0^2+1^2+1.3^2} \approx 1.92$

Score = 0+0+0.27+0.53 = 0.8

Summary and algorithm: Vector space ranking

1. Represent the query as a weighted tf-idf vector
2. Represent each document as a weighted tf-idf vector
3. Compute the cosine similarity score for the query vector and each document vector
4. Rank documents with respect to the query by score
5. Return the top *K* (e.g., *K* = 10) to the user

Resources for today's lecture

- IIR 6.2 – 6.4.3