Introduction to Information Retrieval

http://informationretrieval.org

IIR 6: Scoring, Term Weighting, The Vector Space Model

Hinrich Schütze

Institute for Natural Language Processing, Universität Stuttgart

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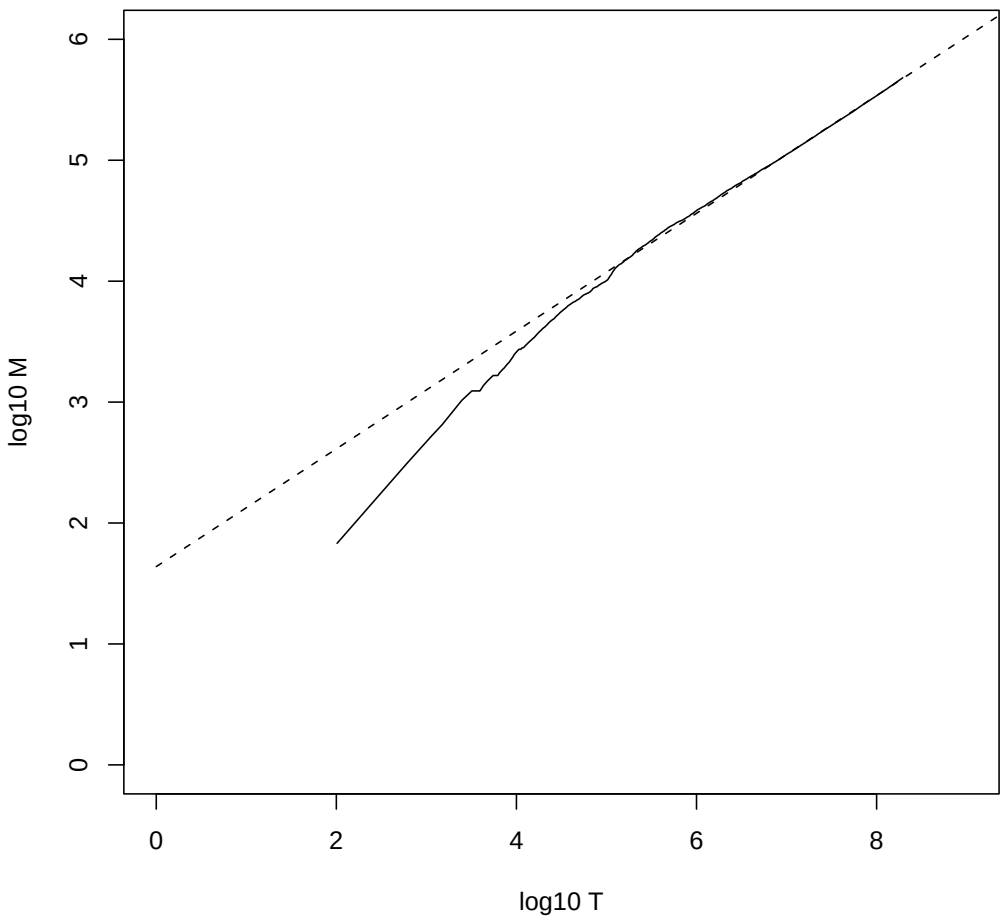
Overview

Outline

Heaps' law

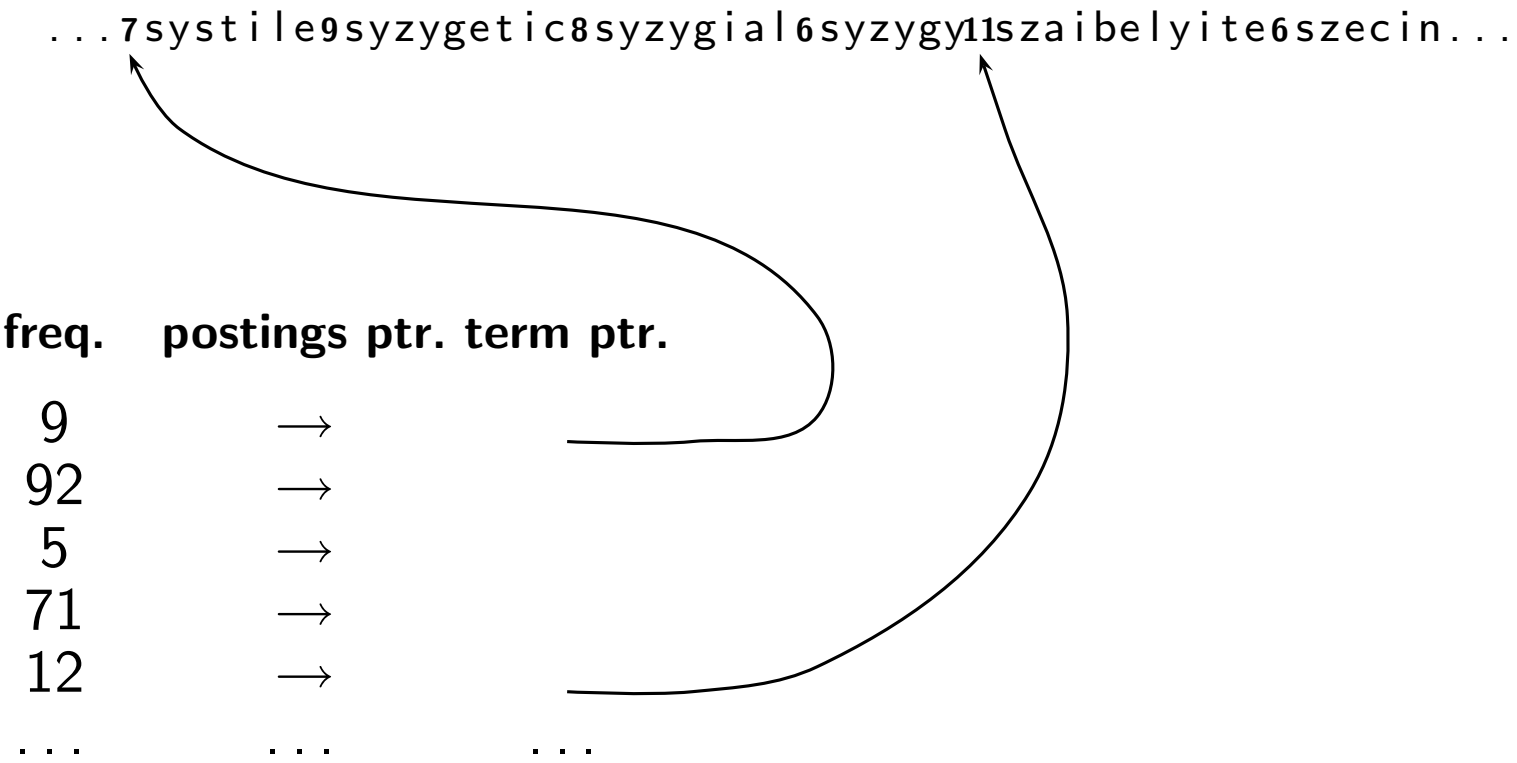


Vocabulary size M as a function of collection size T (number of tokens) for Reuters-RCV1. For these data, the dashed line $\log_{10} M = 0.49 * \log_{10} T + 1.64$ is the best least squares fit. Thus, $M = 10^{1.64} T^{0.49}$ and $k = 10^{1.64} \approx 44$ and $b = 0.49$.

Zipf's law

- Zipf's law: The i^{th} most frequent term has frequency proportional to $1/i$.
- $\text{cf}_i \propto \frac{1}{i}$
- cf is collection frequency: the number of occurrences of the term in the collection.
- So if the most frequent term (*the*) occurs cf_1 times, then the second most frequent term (*of*) has $\text{cf}_1/2$ occurrences, ...
- ...the third most frequent term (*and*) has $\text{cf}_1/3$ occurrences etc.
- About half of all vocabulary terms occur only once in the collection. (hapax legomena)
- Zipf's law is an example of a power law.

Dictionary as a string with blocking



Variable byte (VB) code

- Dedicate 1 bit (high bit) to be a continuation bit c.
- If the gap G fits within 7 bits, binary-encode it in the 7 available bits and set $c = 1$.
- Else: set $c = 0$, encode high-order 7 bits and then use one or more additional bytes to encode the lower order bits using the same algorithm.

Gamma codes for gap encoding

- You can get even more compression with bitlevel code.
- Gamma code is the best known of these.
- Represent a gap G as a pair of length and offset.
- Offset is the gap in binary, with the leading bit chopped off.
- For example $13 \rightarrow 1101 \rightarrow 101$
- Length is the length of offset.
- For 13 (offset 101), this is 3.
- Encode length in unary code: 1110.
- Gamma code of 13 is the concatenation of length and offset: 1110101.

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- Most users are not capable of writing Boolean queries (or they are, but they think it's too much work).
- Most users don't want to wade through 1000s of results.
- This is particularly true of web search.

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- With a ranked list of documents it does not matter how large the retrieved set is.

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- This score measures how well document and query "match".

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- We will look at a number of alternatives for doing this.

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- Always assigns a number between 0 and 1.

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- . . . instead of $|A \cap B|/|A \cup B|$ (Jaccard) for length normalization.

Recall: Binary incidence matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	...
ANTHONY	1	1	0	0	0	1	
BRUTUS	1	1	0	1	0	0	
CAESAR	1	1	0	1	1	1	
CALPURNIA	0	1	0	0	0	0	
CLEOPATRA	1	0	0	0	0	0	
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From now on, we will use the frequencies of terms

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	...
ANTHONY	157	73	0	0	0	1	
BRUTUS	4	157	0	2	0	0	
CAESAR	232	227	0	2	1	0	
CALPURNIA	0	10	0	0	0	0	
CLEOPATRA	57	0	0	0	0	0	
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- For now: bag of words model

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- So we use the log transformation for both term frequency and document frequency.

Examples for idf

Compute idf_t using the formula: $\text{idf}_t = \log_{10} \frac{1{,}000{,}000}{\text{df}_t}$

term	df_t	idf_t
calpurnia	1	
animal	100	
sunday	1000	
fly	10,000	
under	100,000	
the	1,000,000	

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fly	10,000	2
under	100,000	1
the	1,000,000	0

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- Questions about idf?

Collection frequency vs. Document frequency

Word	Collection frequency	Document frequency
INSURANCE	10440	3997
TRY	10422	8760

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- This example suggests that df is better for weighting that cf.

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- Alternative names: tf.idf, tf x idf

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Term, collection and document frequency

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Outline

Binary → count → weight matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	...
ANTHONY	5.25	3.18	0.0	0.0	0.0	0.35	
BRUTUS	1.21	6.10	0.0	1.0	0.0	0.0	
CAESAR	8.59	2.54	0.0	1.51	0.25	0.0	
CALPURNIA	0.0	1.54	0.0	0.0	0.0	0.0	
CLEOPATRA	2.85	0.0	0.0	0.0	0.0	0.0	
MERCY	1.51	0.0	1.90	0.12	5.25	0.88	
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- This is a very sparse vector - most entries are zero.

Queries as vectors

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- Key idea 1: do the same for queries: represent them as vectors in the space
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- proximity = similarity
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- Recall: We're doing this because we want to get away from the you're-either-in-or-out Boolean model.
- Instead: rank more relevant documents higher than less relevant documents

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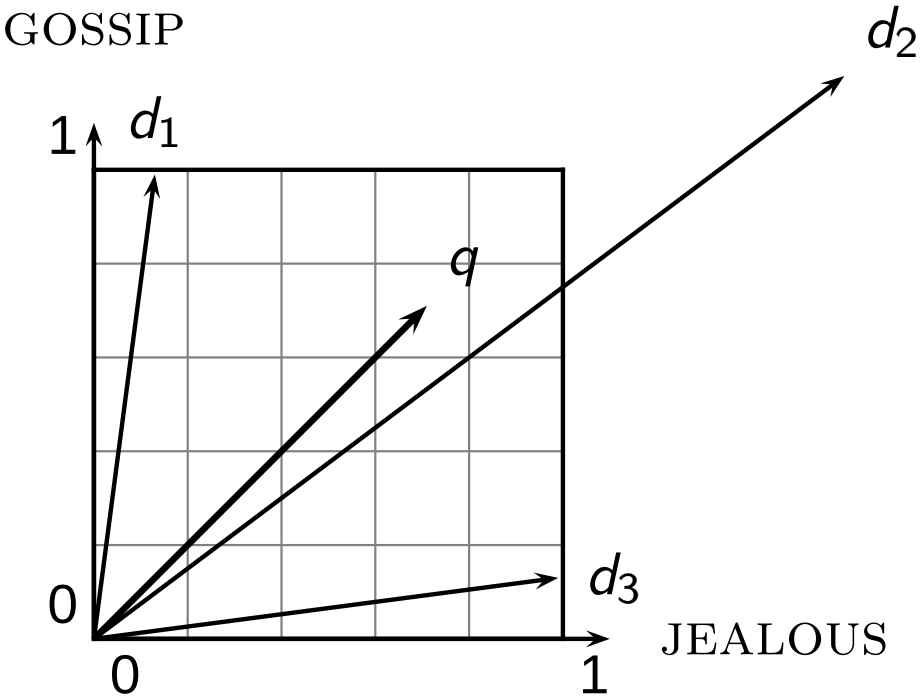
- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths.

Why distance is a bad idea



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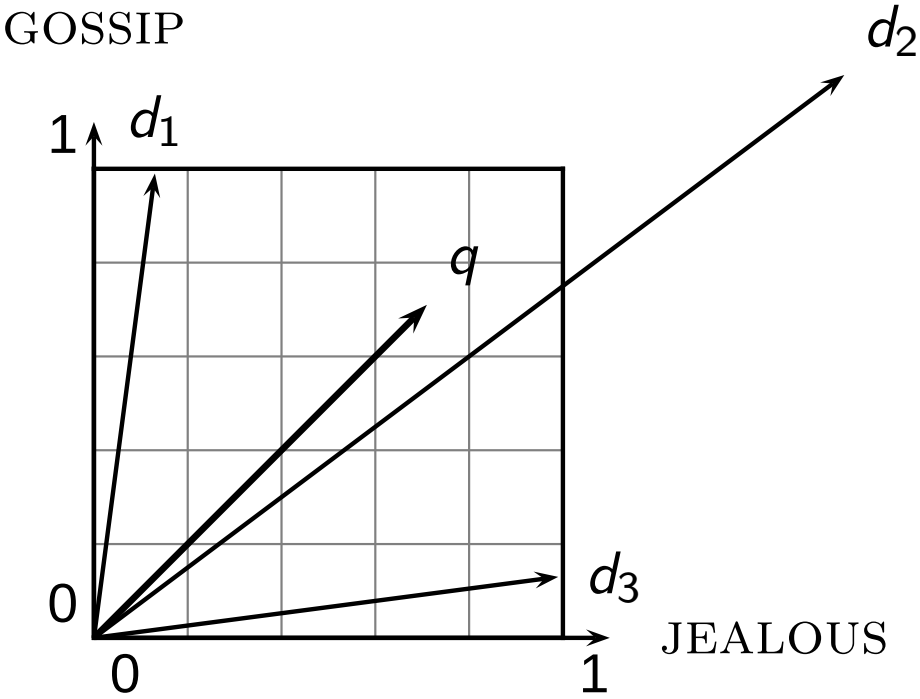

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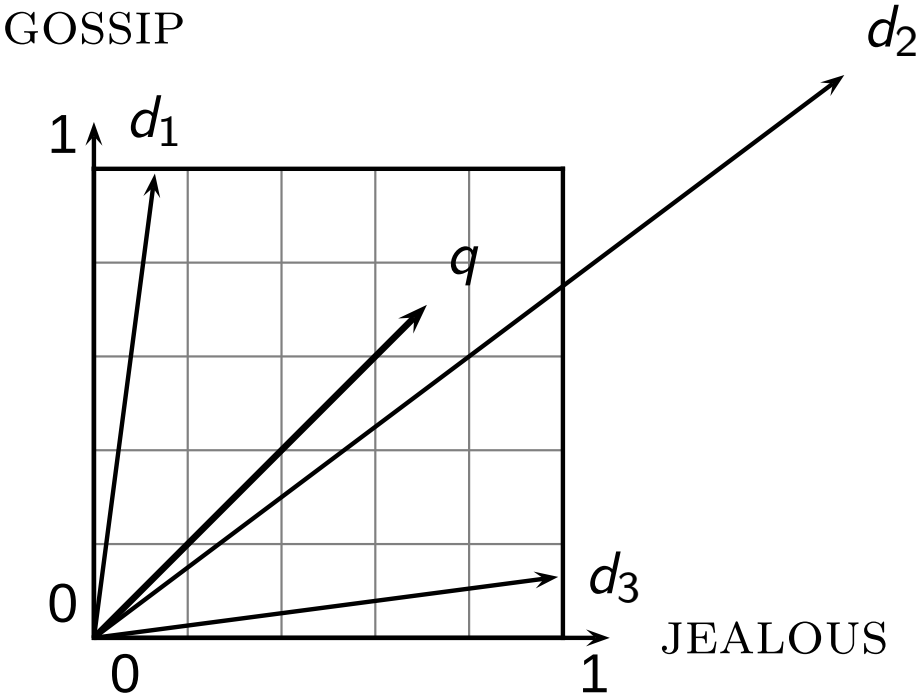

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Questions about basic vector space setup?

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- Rank documents according to angle with query
- Thought experiment: take a document d and append it to itself. Call this document d'.
- "Semantically" d and d' have the same content.
- The angle between the two documents is 0, corresponding to maximal similarity.
- The Euclidean distance between the two documents can be quite large.

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- The following two notions are equivalent.

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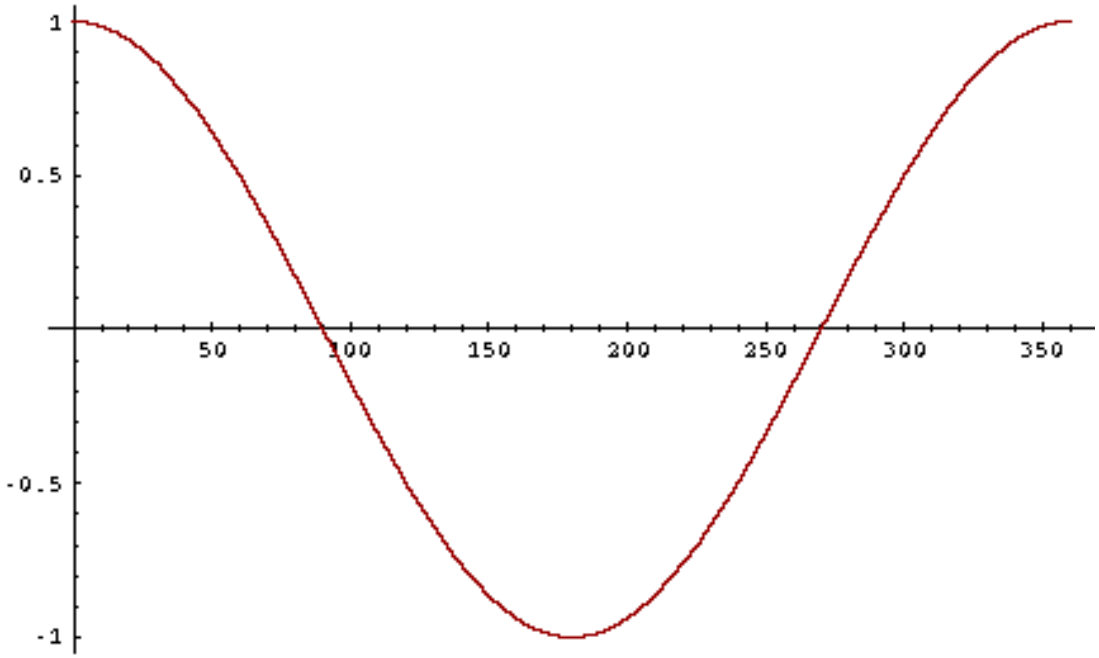
- Cosine is a monotonically decreasing function of the angle for the interval $[0°, 180°]$

Cosine



What about angles $> 180°$?

Length normalization

- How do we compute the cosine?

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- As a result, longer documents and shorter documents have weights of the same order of magnitude.
- Effect on the two documents d and d' (d appended to itself) from earlier slide: they have identical vectors after length-normalization.

Cosine similarity between query and document

$$\cos(\vec{q}, \vec{d}) = \text{SIM}(\vec{q}, \vec{d}) = \frac{\vec{q} \cdot \vec{d}}{|\vec{q}||\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2} \sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

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- This is the cosine similarity of $\vec{q}$ and $\vec{d}$ . . . . . or, equivalently, the cosine of the angle between $\vec{q}$ and $\vec{d}$.

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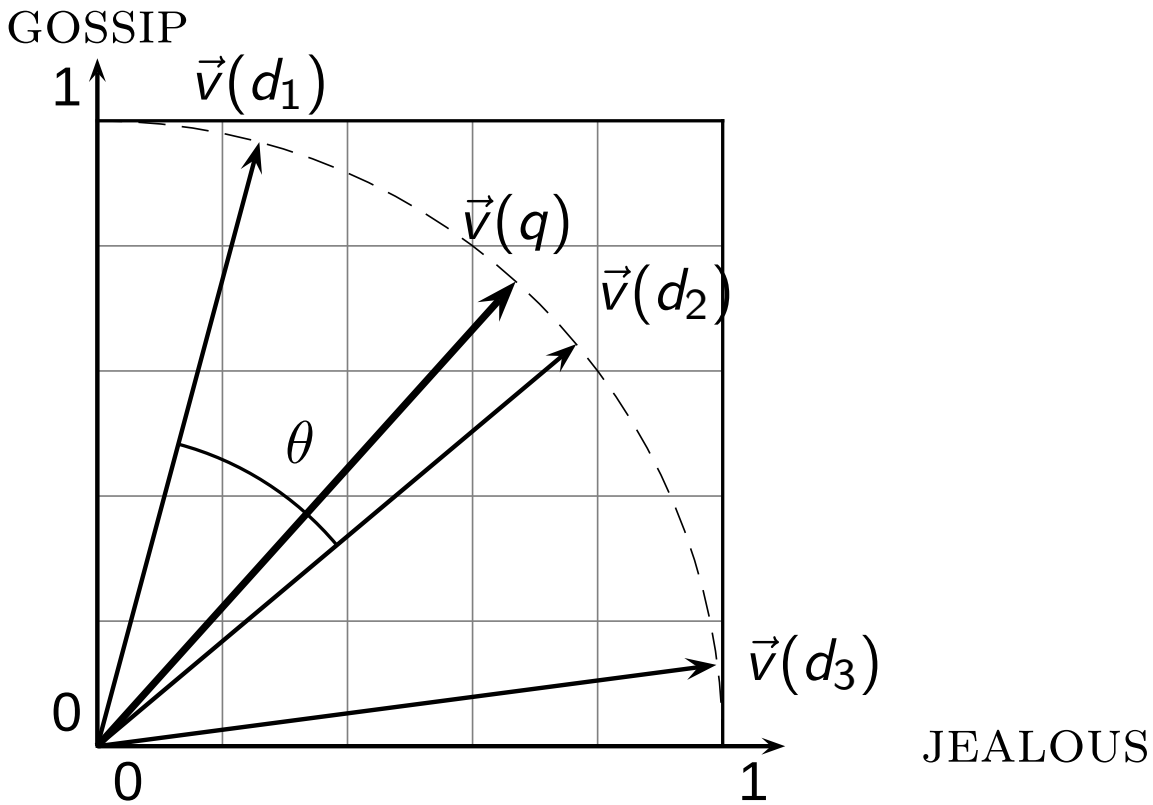
- $\cos(\vec{q}, \vec{d}) = \vec{q} \cdot \vec{d} = \sum_i q_i \cdot d_i$ (if $\vec{q}$ and $\vec{d}$ are length-normalized).

Cosine similarity illustrated



Cosine: Example

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(To simplify this example, we don't do idf weighting.)

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- cos(PaP,WH) ≈ 0.69
- Why do we have cos(SaS,PaP) $>$ cos(SAS,WH)?

Computing the cosine score

CosineScore(q)

```
1  float Scores[N] = 0
2  float Length[N]
3  for each query term t
4  do calculate w_{t,q} and fetch postings list for t
5      for each pair(d, tf_{t,d}) in postings list
6      do Scores[d] += w_{t,d} × w_{t,q}
7  Read the array Length
8  for each d
9  do Scores[d] = Scores[d]/Length[d]
10 return Top K components of Scores[]
```

Components of tf-idf weighting

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2+w_2^2+\ldots+w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times \mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N-\mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/\mathit{CharLength}^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1+\log(\mathrm{tf}_{t,d})}{1+\log(\mathrm{ave}_{t \in d}(\mathrm{tf}_{t,d}))}$				

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Best known combination of weighting options

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Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2+w_2^2+\ldots+w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times \mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N-\mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/\mathit{CharLength}^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1+\log(\mathrm{tf}_{t,d})}{1+\log(\mathrm{ave}_{t \in d}(\mathrm{tf}_{t,d}))}$				

Default: no weighting

tf-idf example

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- Example query: "best car insurance"
- Example document: "car insurance auto insurance"

tf-idf example: ltn.lnc

Query: "best car insurance". Document: "car insurance auto insurance".

word	query					document				product
	tf-raw	tf-wght	df	idf	weight	tf-raw	tf-wght	weight	n'lized	
auto										
best										
car										
insurance										

Key to columns: tf-raw: raw (unweighted) term frequency, tf-wght: logarithmically weighted term frequency, df: document frequency, idf: inverse document frequency, weight: the final weight of the term in the query or document, n'lized: document weights after cosine normalization, product: the product of final query weight and final document weight

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Query: "best car insurance". Document: "car insurance auto insurance".

word	query					document				product
	tf-raw	tf-wght	df	idf	weight	tf-raw	tf-wght	weight	n'lized	
auto	0									
best	1									
car	1									
insurance	1									

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auto	0					1				
best	1					0				
car	1					1				
insurance	1					2				

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auto	0	0				1	1			
best	1	1				0	0			
car	1	1				1	1			
insurance	1	1				2	1.3			

Key to columns: tf-raw: raw (unweighted) term frequency, tf-wght: logarithmically weighted term frequency, df: document frequency, idf: inverse document frequency, weight: the final weight of the term in the query or document, n'lized: document weights after cosine normalization, product: the product of final query weight and final document weight

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word	query					document				product
	tf-raw	tf-wght	df	idf	weight	tf-raw	tf-wght	weight	n'lized	
auto	0	0	5000			1	1			
best	1	1	50000			0	0			
car	1	1	10000			1	1			
insurance	1	1	1000			2	1.3			

Key to columns: tf-raw: raw (unweighted) term frequency, tf-wght: logarithmically weighted term frequency, df: document frequency, idf: inverse document frequency, weight: the final weight of the term in the query or document, n'lized: document weights after cosine normalization, product: the product of final query weight and final document weight

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car	1	1	10000	2.0		1	1			
insurance	1	1	1000	3.0		2	1.3			

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best	1	1	50000	1.3	1.3	0	0			
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best	1	1	50000	1.3	1.3	0	0	0	0	
car	1	1	10000	2.0	2.0	1	1	1	0.52	
insurance	1	1	1000	3.0	3.0	2	1.3	1.3	0.68	

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$\sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92$
$1/1.92 \approx 0.52$
$1.3/1.92 \approx 0.68$

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best	1	1	50000	1.3	1.3	0	0	0	0	0
car	1	1	10000	2.0	2.0	1	1	1	0.52	1.04
insurance	1	1	1000	3.0	3.0	2	1.3	1.3	0.68	2.04

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car	1	1	10000	2.0	2.0	1	1	1	0.52	1.04
insurance	1	1	1000	3.0	3.0	2	1.3	1.3	0.68	2.04

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Final similarity score between query and document: $\sum_i w_{qi} \cdot w_{di} = 0 + 0 + 1.04 + 2.04 = 3.08$

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best	1	1	50000	1.3	1.3	0	0	0	0	0
car	1	1	10000	2.0	2.0	1	1	1	0.52	1.04
insurance	1	1	1000	3.0	3.0	2	1.3	1.3	0.68	2.04

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Questions?

Summary: Ranked retrieval in the vector space model

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- Rank documents with respect to the query

Summary: Ranked retrieval in the vector space model

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity between the query vector and each document vector
- Rank documents with respect to the query
- Return the top K (e.g., $K = 10$) to the user

Resources

- Chapters 6 and 7 of IIR

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- Okapi BM25 (a state-of-the-art weighting method, 11.4.3 of IIR)