Color Spaces and Color-Difference Equations

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1 Color Spaces

A color-order system is a conceptual system of organized color perceptions. The Munsell Color System was the first color-order system developed by Munsell in 1905 as a teaching aid for art students [9]. His goal was to have both a numerical system and a physical exemplification, achieved via the *Atlas of the Munsell Colors* [10]. A color-order system exemplification is a physical system that depicts a color-order system. His guiding principle was equality of visual spacing. The original Atlas has undergone considerable refinement and development based on extensive visual experimentation resulting in the current specification, the *Munsell System*, and its various exemplifications. During late 1930s, a subcommittee of the Optical Society of America performed further visual experiments to improve the visual spacing between color samples. More sophisticated color matching experimental techniques were used [11], totaling over three million observations in the final specifications [12].

A Munsell notation is defined as H V/C for hue (H), value (V), and chroma (C). The Munsell system is cylindrical in nature with a central lightness axis (Munsell value) surrounded by chromatic planes arranged in a hue circle. For a given hue, the colors have equal visual spacing. However, between hues, the more chromatic colors are further apart than less chromatic colors are. Between 1947 and 1974, the Optical Society of America developed a new color-order system, called the OSA Uniform Color Scales (OSA-UCS), that alleviated this deficiency. Unfortunately, the OSA-UCS is very complex and very difficult to sample colors at constant hue or chroma, thus greatly limiting its usage.

The International Commission on Illumination (Commission International de l''Eclairage, CIE) standardized a method of specifiying the color of illuminants and materials by tristimulus values X, Y, Z:

$$\begin{aligned} X &= k\int \Phi(\lambda)\bar{x}(\lambda)d\lambda \\ Y &= k\int \Phi(\lambda)\bar{y}(\lambda)d\lambda \\ Z &= k\int \Phi(\lambda)\bar{z}(\lambda)d\lambda \end{aligned} \tag{1}$$

where $\Phi(\lambda)$ describes the spectral power of the stimulus and $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are the color-matching functions of the 1931 CIE standard observer. For an illuminant, $\Phi(\lambda) = S(\lambda)$, the relative spectral power of the illuminant. For a reflecting material, $\Phi(\lambda) = S(\lambda)R(\lambda)$ where $S(\lambda)$ is the relative power of an illuminant and $R(\lambda)$ is the reflection factor of the

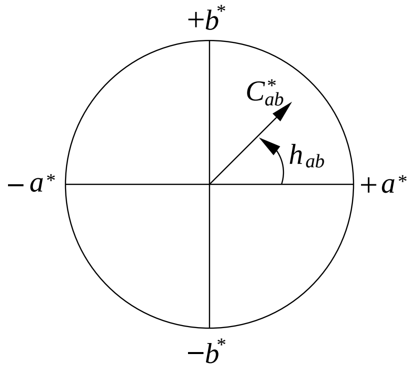

Figure 1: CIELAB chroma C^*_{ab} and hue angle h_{ab}.

material. For a transparent material, $\Phi(\lambda) = S(\lambda)T(\lambda)$ where $T(\lambda)$ is the transmission factor of the material. By convention, Y is assigned the value 100 for a *perfect reflecting diffuser*, i.e., an ideal nonflourescent white reflecting 100% at all wavelength. For a perfect colorless material transmitting 100% at all wavelength (i.e., no sample at all), Y is also assigned the value 100.

The spacing of the colors in the XYZ space is not perceptually uniform. The XYZ space can be transformed to a more nearly uniform CIE 1976 $L^*a^*b^*$ (CIELAB) color space. The transformation equations for CIELAB are:

$$\begin{aligned}
L^* &= 116\left[f\left(\frac{Y}{Y_n}\right) - \frac{16}{116}\right] \\
a^* &= 500\left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right] \\
b^* &= 200\left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right] \\
f(w) &= \begin{cases} w^{1/3} & \text{if } w > 0.008856 \\ 7.787w + \dfrac{16}{116} & \text{otherwise} \end{cases} \\
C^*_{ab} &= (a^{*2} + b^{*2})^{1/2} \\
h_{ab} &= \tan^{-1}\left(\frac{b^*}{a^*}\right)
\end{aligned} \tag{2}$$

where X_n, Y_n, and Z_n are the tristimulus values of the reference white, L^* denotes lightness, a^* and b^* denote chromaticity, C^*_{ab} denotes chroma, and h_{ab} denotes hue. The hue angle is measured in degrees starting with $h_{ab} = 0$ in the $+a^*$ axis direction and increasing counterclockwise (Fig. 1). That is,

$$\begin{aligned}
0^\circ < h_{ab} < 90^\circ \quad & \text{if } a^*, b^* > 0 \\
90^\circ < h_{ab} < 180^\circ \quad & \text{if } a^* < 0, b^* > 0 \\
180^\circ < h_{ab} < 270^\circ \quad & \text{if } a^*, b^* < 0 \\
270^\circ < h_{ab} < 360^\circ \quad & \text{if } a^* > 0, b^* < 0
\end{aligned} \tag{3}$$

The CIELAB equations were derived such that the illuminant is always at $L^* = 100$, $a^* = 0$, $b^* = 0$ ([2], page 68). So, the illuminant is the reference white. Table 1 lists the tristimulus

Table 1: Tristimulus values of common illuminants and observer combinations. The values are normalized to $Y = 100$.

illuminant	observer	X	Y	Z
D65	2°	95.047	100.000	108.883
	10°	94.811	100.000	107.304
D60	2°	96.422	100.000	82.521
	10°	96.720	100.000	81.427

values of common illuminants and observer combinations [2]. Industries such as paints, plastics, and textiles have adopted D65. The graphic arts and computer industries use D50.

The inverse transformation from CIELAB to XYZ is given by:

$$\begin{aligned}
a^* &= C^*_{ab} \cos h_{ab} \\
b^* &= C^*_{ab} \sin h_{ab} \\
X &= X_n\, f^{-1}\left(\frac{L^* + 16}{116} + \frac{a^*}{500}\right) \\
Y &= Y_n\, f^{-1}\left(\frac{L^* + 16}{116}\right) \\
Z &= Z_n\, f^{-1}\left(\frac{L^* + 16}{116} - \frac{b^*}{200}\right) \\
f^{-1}(w) &= \begin{cases} w^3 & \text{if } w > 0.008856^{1/3} \\ \frac{1}{7.787}\left(w - \frac{16}{116}\right) & \text{otherwise} \end{cases}
\end{aligned} \tag{4}$$

2 Color-Difference Equations

Total color difference ΔE^*_{ab} from a reference color (L^*_0, a^*_0, b^*_0) to a target color (L^*_1, a^*_1, b^*_1) in the CIELAB space is given by:

$$\Delta E^*_{ab} = \left[(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2\right]^{1/2} \tag{5}$$

where

$$\begin{aligned}
\Delta L^* &= L^*_1 - L^*_0 \\
\Delta a^* &= a^*_1 - a^*_0 \\
\Delta b^* &= b^*_1 - b^*_0
\end{aligned} \tag{6}$$

or

$$\Delta E^*_{ab} = \left[(\Delta L^*)^2 + (\Delta C^*_{ab})^2 + (\Delta H^*_{ab})^2\right]^{1/2} \tag{7}$$

where

$$\begin{aligned}
\Delta C^*_{ab} &= C^*_{ab,1} - C^*_{ab,0} = (a^{*2}_1 + b^{*2}_1)^{1/2} - (a^{*2}_0 + b^{*2}_0)^{1/2} \\
\Delta H^*_{ab} &= [(\Delta E^*_{ab})^2 - (\Delta L^*)^2 - (\Delta C^*_{ab})^2]^{1/2}
\end{aligned} \tag{8}$$

Stokes and Brill [16] derived a direct and efficient method of computing ΔH^*_{ab}:

$$\Delta H^*_{ab} = s\,[2(Q - a^*_0 a^*_1 - b^*_0 b^*_1)]^{1/2} \tag{9}$$

where

$$\begin{aligned} Q &\equiv C^*_{ab,0} C^*_{ab,1} = [(a^{*2}_0 + b^{*2}_0)(a^{*2}_1 + b^{*2}_1)]^{1/2} \\ s &= \begin{cases} 1 & \text{if } a^*_0 b^*_1 > a^*_1 b^*_0 \\ -1 & \text{otherwise} \end{cases} \end{aligned} \tag{10}$$

The CIE 1994 color-difference equation (CIE94) is more perceptually uniform than the ΔE^*_{ab} [2]. CIE94 is given by:

$$\begin{aligned} \Delta E^*_{94} &= \left[\left(\frac{\Delta L^*}{k_L S_L} \right)^2 + \left(\frac{\Delta C^*_{ab}}{k_C S_C} \right)^2 + \left(\frac{\Delta H^*_{ab}}{k_H S_H} \right)^2 \right]^{1/2} \\ S_L &= 1 \\ S_C &= 1 + 0.045\,\bar{C}^*_{ab} \\ S_H &= 1 + 0.015\,\bar{C}^*_{ab} \\ k_L &= k_C = k_H = 1 \text{ for reference conditions} \\ \bar{C}^*_{ab} &= \sqrt{C^*_{ab,0} C^*_{ab,1}} \end{aligned} \tag{11}$$

Other CIELAB-based color-difference equations include CMC and BFD [2]. A color-difference unit based on the uniform chromaticity scale of Hunter (1942) was designated the National Bureau of Standards (NBS) unit (or modified Judd). This system is rarely used. (Note: [4] uses the Godlove equation [3] for color clustering and image retrieval).

Recent test results are summarized below:

- [6]
  Test CIELAB, CMC, BFD, and CIE94. Mean color difference is 3 ΔE^*_{ab}. CMC, BFD, CIE94 are more accurate than CIELAB. BFD is the most accurate.

- [5]
  Test CIELAB, CMC, BFD, and CIE94. Mean color difference is 13 ΔE^*_{ab}. CIELAB is most accurate followed closely by CIE94.

- [7]
  Test color-difference accuracy of Adams-Nickerson formula, modified Judd (NBS) formula, CIELUV, CIE94. CIE94 is most accurate.

- [8]
  Based on classical and recent datasets. CMC, BFD, and CIE94 are more uniform than CIELUV and CIELAB. CIE94 is consistently better than CMC, and is better than BDF about half the time.

- [17]
  Large-size printed images. Test perceptibility prediction of color-difference equations. ΔE^*_{ab} and ΔE^*_{94}(2:1:1) describe color difference between two images more accurately than does ΔE^*_{94}(1:1:1). The perceptibility threshold is 1.95 ΔE^*_{ab}.

- [14]
  Present images in [17] on a CRT monitor. Test perceptibility and acceptability of CIELAB, CMC, BFD, CIE94, CIELAB2000. ΔE^*_{ab} for perceptibility is about 2.2 and 4.5 for acceptability. There is not much difference between different color-difference formulae using optimized weights. All formulae performed slightly better than CIELAB.

3 RGB Color Space

According to [13], the transformation of RGB values of NTSC CRT to XYZ values is given by:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.607 & 0.174 & 0.201 \\ 0.299 & 0.587 & 0.114 \\ 0.000 & 0.066 & 1.117 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \tag{12}$$

Here, $0 \leq R, G, B \leq 1$. The white color has RGB values of (1, 1, 1) and XYZ values of (0.982, 1.000, 1.183). This tristimulus values does not correspond to those of D65 and D60. It is probably the tristimulus values of the phosphors at maximum voltage. Note that the matrix values vary according to the phosphors characteristics of the CRT monitor.

The inverse transform is given by:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.910 & -0.533 & -0.288 \\ -0.985 & 2.000 & -0.028 \\ 0.058 & -0.118 & 0.896 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{13}$$

Stokes et al. [1, 15] proposed a standard default color space for the internet called the *sRGB*. Transformation from *sRGB* to XYZ is as follows:

$$R' = R_8/255.0, \quad G' = G_8/255.0, \quad B' = B_8/255.0 \tag{14}$$

Here, $0 \leq R_8, G_8, B_8 \leq 255$. Then,

$$R_s = \begin{cases} R'/12.92 & \text{if } R' \leq 0.03928 \\ \left[\dfrac{R' + 0.055}{1.055}\right]^{2.4} & \text{otherwise} \end{cases} \tag{15}$$

Similarly for G_s and B_s. Finally,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix} \begin{bmatrix} R_s \\ G_s \\ B_s \end{bmatrix} \tag{16}$$

The white color has $R_sG_sB_s$ values of (1, 1, 1) and XYZ values of (0.9505, 1.0000, 1.0890), which is the tristimulus values of D65 illuminant.

The inverse transform is

$$\begin{bmatrix} R_s \\ G_s \\ B_s \end{bmatrix} = \begin{bmatrix} 3.2410 & -1.5374 & -0.4986 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0570 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{17}$$

$$R' = \begin{cases} 12.92 R_s & \text{if } R_s \leq 0.00304 \\ 1.055 R_s^{1.0/2.4} - 0.055 & \text{otherwise} \end{cases} \tag{18}$$

Similarly for G' and B'. Finally,

$$R_8 = 255.0R', \quad G_8 = 255.0G', \quad B_8 = 255.0B' \tag{19}$$

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