# Modeling Torsion of Blood Vessels in Surgical Simulation and Planning

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**Abstract.** This paper proposes a hybrid approach for modeling torsion of blood vessels that undergo deformation and joining. The proposed model takes 3D mesh of the blood vessel as input. It first fits a generalized cylinder to extract the blood vessel's medial axis. Then, it uses rotation minimizing frame as a reference to model and measure the torsion of blood vessel after deformation. In general, the proposed approach can incorporate any kind of deformation algorithms. In our experiments, differential geometry method is used as an example. The test results show that our algorithm can correctly and effectively evaluate the amount of torsion caused by blood vessel deformation. In addition, it can also determine the configuration of the blood vessel with minimum torsion.

**Keywords.** Blood vessel torsion, surgical simulation and planning, generalized cylinder, rotation minimizing frame

## Introduction

Operations on 3D tubular models such as deformation, cutting, and joining are essential for the simulation of surgeries on blood vessels. Traditional approach models blood vessels using parametric models such as splines and generalized cylinders. These parametric models are easy to manipulate. But they usually lack surface details and assume no change in cross-sectional shape during deformation. These conditions are invalid in operations on the great arteries, where cutting and joining can change their cross-sectional shapes. Parametric models are thus inappropriate for these applications.

An alternative approach is to model blood vessels as 3D meshes. When a blood vessel is operated on, it can undergo stretching, bending, and torsion. In surgical planning, these deformations should be modeled and measured to assist the surgeon in evaluating various surgical options. Stretching and bending are easy to model in 3D mesh, whereas modeling of torsion is nontrivial.

To overcome the shortcomings of these existing methods, this paper proposes a novel approach for modeling blood vessels by integrating 3D mesh, generalized cylinder, and Cosserat rod theory. This hybrid approach allows the model to capture surface details and to vary cross-sectional shape during deformation. Moreover, torsion of the model can be computed in a physically correct manner using Cosserat rod theory. Then, the model's

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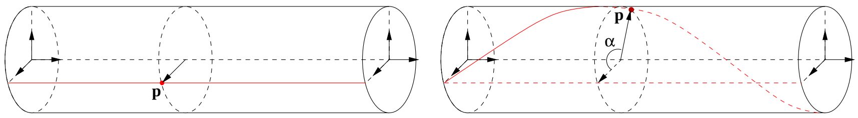


(a) (b)

**Figure 1.** Material torsion angle $\alpha$.

configuration that minimizes torsion can also be determined. This approach is useful for surgical simulation of blood vessel joining that requires minimum torsion.

## 1. Modeling Material Torsion

The Cosserat rod model is a well-known approach for modeling tubular objects whose length is much greater than the diameter [1]. In this approach, the medial axis of a tubular object is modeled by a parameterized curve $\mathbf{m}(s)$, $0 \leq s \leq 1$. Attached to each point on the medial axis is the *director frame* $\{\mathbf{T}(s), \mathbf{U}(s), \mathbf{V}(s)\}$ formed by three orthonormal basis vectors. $\mathbf{T}(s)$ is the normal of the tubular object's cross-sectional plane $\mathcal{X}_s$ at $s$. Assuming the object does not shear, $\mathbf{T}(s)$ is tangential to the medial axis, i.e., $\mathbf{T}(s) = \mathbf{m}'(s)/\|\mathbf{m}'(s)\|$. $\mathbf{U}(s)$ is in $\mathcal{X}_s$ and points towards a material point on the surface of the object. $\mathbf{V}(s) = \mathbf{T}(s) \times \mathbf{U}(s)$ points to another material point. The directors must be differentiable over $s$ such that the material points represented by $\mathbf{U}$ and $\mathbf{V}$ form continuous material lines.

Two kinds of torsions can be computed on a tubular object that undergoes deformation [2]. The *Frenet torsion* measures the twisting of the space curve that represents the medial axis. The *material torsion* measures the rotation of the material line about the medial axis, which is the focus of this paper. To measure the rotation angle of the material line, a reference line on the object's surface must be defined. This reference line has the same shape as the medial axis.

One way to define the reference line is to use Frenet frame of the medial axis, whose three axis vectors are the tangent, principle normal and bi-normal at $s$. However, Frenet scheme is ill-defined at an inflection point, where the curvature and normal change sign. This results in the undesirable flipping of the directions of the Frenet frames.

Another way to define the reference line is to use rotation minimizing frame (RMF) of the medial axis [3,4]. An RMF is a moving frame along the medial axis that minimizes the amount of rotation of the frame. It does not have the undesirable property of the Frenet frame. So, in this paper, we apply RMF to define the reference line.

It is difficult to compute the exact RMF for a general spline curve [3]. A projection method [4] and a rotation method [5] can be used to approximate discrete RMF. These methods have second-order global approximation error [6]. Wang et al. [3] presented a simple and efficient algorithm to approximate RMF, namely double reflection method. Their method has fourth-order global approximation error, and is thus more accurate than the first two. Thus, we adopt the double reflection method to compute RMF.

Without loss of generality, we define the initial configuration of the blood vessel such that the directors at each point are aligned to the corresponding RMF. After deformation, the directors may be rotated about the medial axis. This rotation angle $\alpha(s)$ is the angle of rotation between the directors and the medial axis's RMF (Fig. 1). Then, the *material torsion* is the first derivative of $\alpha(s)$, i.e., $\alpha'(s)$.

## 2. The Proposed Model

To model a blood vessel's material torsion due to deformation, we need to first define its medial axis (Section 2.1). The medial axis is extracted from the 3D mesh by fitting a generalized cylinder (GC) to the mesh. After fitting, correspondence between mesh vertices and the GC can be established. Each mesh vertex is associated with a position on the medial axis and an angle representing its cross-sectional orientation (Section 2.2). A differential geometry method [7] is applied to deform the 3D mesh model. For any deformed configuration, the correspondence between the mesh vertices and the GC is used to recover the deformed medial axis, and the material torsion can be computed. Based on the computed torsion energy, an optimization algorithm is then applied to determine the model's configuration that minimizes torsion (Section 2.3).

### *2.1. Medial Axis*

Let $\mathbf{p}_i$ denote a point on the surface of the 3D tubular object, and $\mathbf{m}(s_i)$ denote the projection of $\mathbf{p}_i$ on the medial axis. Then, $\mathbf{m}(s)$ would be the centroid of all the surface points that project to it. In the case of a discrete 3D mesh model of the tubular object, the points $\mathbf{p}_i$ are taken as the mesh vertices, and $\mathbf{m}(s)$ is defined in terms of the mesh vertices $\mathbf{p}_i$ whose projections are close to it:

$$\mathbf{m}(s) = \sum_i w_i(s)\mathbf{p}_i \,/\, \sum_i w_i(s). \tag{1}$$

The weights $w_i(s)$ are inversely related to the distance between $\mathbf{m}(s)$ and $\mathbf{m}(s_i)$.

### *2.2. Fitting Generalized Cylinder*

The algorithm for fitting a generalized cylinder to the 3D mesh is adapted from [8]. It consists of two steps: (1) fitting the medial axis and (2) fitting the radius function. The cost $C_m(\mathbf{m})$ of fitting the medial axis is derived from Eq. (1):

$$C_m(\mathbf{m}) = \frac{1}{2}\int_0^1 \left\| \mathbf{m}(s) - \frac{\sum_i w_i(s)\mathbf{p}_i}{\sum_i w_i(s)} \right\|^2 ds + \frac{\lambda}{2}\int_0^1 \|\nabla \mathbf{m}(s)\|^2 ds. \tag{2}$$

The second term is a regularization term and $\lambda$ is the regularization weight. Differentiating the cost function yields the gradient decent equation for medial axis fitting:

$$\Delta \mathbf{m} = -\sum_i w_i(s)(\mathbf{m}(s) - \mathbf{p}_i)\left(\sum_i w_i(s)\right)^{-1} - \lambda\, \Delta_s \mathbf{m}(s) \tag{3}$$

where $\Delta_s \mathbf{m}(s)$ is the Laplacian of $\mathbf{m}(s)$.

The cost function $C_r(r)$ for fitting the radius function $r(s)$ is given by:

$$C_r(r) = \frac{1}{2}\int_0^1 \left[ r(s) - \frac{1}{N_s}\sum_i \|\mathbf{m}(s) - \mathbf{p}_i\| \right]^2 ds + \frac{\mu}{2}\int_0^1 (r'(s))^2 ds \tag{4}$$

where $\mathbf{p}_i$ are the mesh vertices whose projections to the medial axis are close to $\mathbf{m}(s)$, and $N_s$ the number of such $\mathbf{p}_i$. Differentiating the cost function yields the gradient descent equation for radius fitting:

$$\Delta r = -r(s) + \frac{1}{N_s}\sum_i \|\mathbf{m}(s) - \mathbf{p}_i\| - \mu\, r''(s). \tag{5}$$

After fitting the generalized cylinder to the mesh model, the RMFs can be computed [3], which are regarded as the initial director frames $\{\mathbf{T}(s), \mathbf{U}(s), \mathbf{V}(s)\}$. The mesh vertices can then be parameterized as $(s, \theta)$. For mesh vertex $\mathbf{p}_i$ that projects to the point $\mathbf{m}(s_i)$ on the medial axis, its parameter $s = s_i$. Its parameter $\theta$ is given by the rotation angle of the vector $\mathbf{p}_i - \mathbf{m}(s)$ about $\mathbf{T}(s)$ measured from the $\mathbf{U}(s)$ axis.

### *2.3. Measurement of Deformation*

Deformation of the 3D mesh of tubular object can be achieved using a variety of algorithms such as free-form deformation (FFD), differential geometry (DG) method, mass-spring model, and Finite Element Method. The latter two methods model forces acting on the object which are opposed by internal forces. On the other hand, FFD and DG deal with geometric properties and do not model forces. In particular, geometric constraints can be easily incorporated into DG formulation. Therefore, a DG method called Laplacian deformation [7] is used in our application to perform mesh deformation. In joining two blood vessel models, this method requires the user to specify a pair of corresponding anchor points on the two models. Inappropriate anchor points can lead to torsion of the blood vessels. In contrast, our torsion modeling technique can automatically deduce the configuration that minimizes torsion without requiring user input of anchor points.

In our formulation, torsion due to deformation is computed as follows. For the 3D mesh of the tubular object before deformation, the director frames $\{\mathbf{T}, \mathbf{U}, \mathbf{V}\}$ are the same as the RMF. So, the rotation angles $\alpha(s)$ are equal to 0. That is, the material torsions $\alpha'(s)$ before deformation are 0, which are the expected values.

After deformation, the medial axis of the deformed mesh is recovered by computing the $\mathbf{m}(s)$ that satisfy the following equation:

$$\sum_i w_i(s)[\mathbf{m}(s) - \mathbf{p}_i]\left(\sum_i w_i(s)\right)^{-1} + \lambda\, \Delta_s \mathbf{m}(s) = 0, \tag{6}$$

which is derived by setting $\Delta\mathbf{m}$ in Eq. (3) to $0$.

With the deformed medial axis, the RMF along the medial axis can also be recovered using the double reflection method [3]. Now, the mesh vertex $\mathbf{p}_i$ has a new parameterization $(s_i, \theta_i)$ according to the medial axis and RMF of the deformed mesh. Let us denote the original parameterization of $\mathbf{p}_i$ before deformation as $(s_i^0, \theta_i^0)$. Then, the angle $\alpha(s)$ after deformation can be computed as:

$$\alpha(s) = \frac{1}{N_s}\sum_i (\theta_i - \theta_i^0) \tag{7}$$

where $i$ refers to the indices of points whose projections to the medial axis, $\mathbf{m}^0(s_i^0)$ before deformation and $\mathbf{m}(s_i)$ after deformation, are close to $\mathbf{m}^0(s)$ and $\mathbf{m}(s)$ respectively, and $N_s$ is the number of such points $\mathbf{p}_i$. Then, the torsion energy $E_\tau$ can be computed:

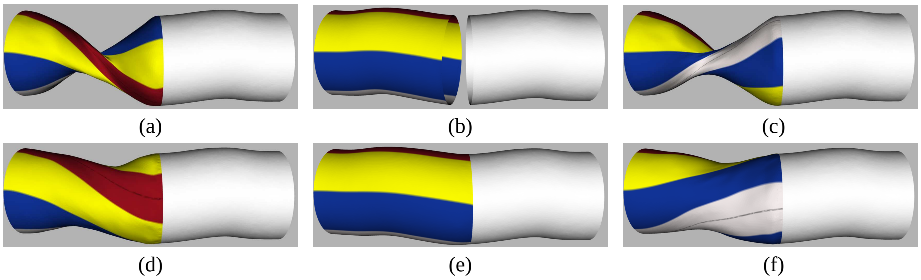

**Figure 2.** Test case 1: Joining of two relatively straight blood vessel models. The left blood vessel is shaded in color to visualize torsion. (a, c) Large amount of torsion. (b) Initial configuration. (d, f) Medium torsion. (e) Minimum torsion configuration found by the algorithm.

$$E_\tau = \int_0^1 (\alpha'(s))^2 ds. \tag{8}$$

Given a deformed 3D mesh, the amount of deformation in terms of torsion can be measured according to the equations described above. So, a straightforward optimization algorithm can be devised to determine the configuration of 3D mesh that satisfies the user's inputs and incurs the smallest amount of torsion.

## 3. Experimental Results

Two test cases of blood vessel joining were performed to evaluate the algorithm's performance. In the first case, two relatively straight blood vessels were joined (Fig. 2). The right end of the left blood vessel was free to rotate before joining. The algorithm correctly computed the amount of torsion and determined the deformed configuration with minimum torsion (Fig. 2(e)).

The second test case joined the aorta to its root (Fig. 3). The aorta model was highly curved at the aortic arch. The descending aorta was fixed while the ascending aorta and the aortic arch were free to move. The end of the ascending aorta was moved to join with the aortic root. The algorithm correctly determined the deformed configuration with the smallest amount of torsion (Fig. 3(e)).

For illustration, Figure 4 plots the torsion energy at various amount of rotation of the free end with respect to the blood vessels initial configuration. It shows that the torsion energy behaves as a quadratic function of the rotation angle. The configuration with minimum torsion is labeled 0, while the others are labeled with $+$ or $-$ sign according to the rotation angle.

## 4. Conclusion

This paper presented a hybrid approach for modeling torsion of blood vessels that undergo deformation and joining. Test results show that our algorithm can correctly model and measure the amount of torsion due to blood vessel deformation. It also determines the configuration of the blood vessels with minimum torsion. In this way, two blood vessels can be joined with minimum torsion without requiring the user to specify accurate corresponding anchor points, thus facilitating surgical planning and simulation.

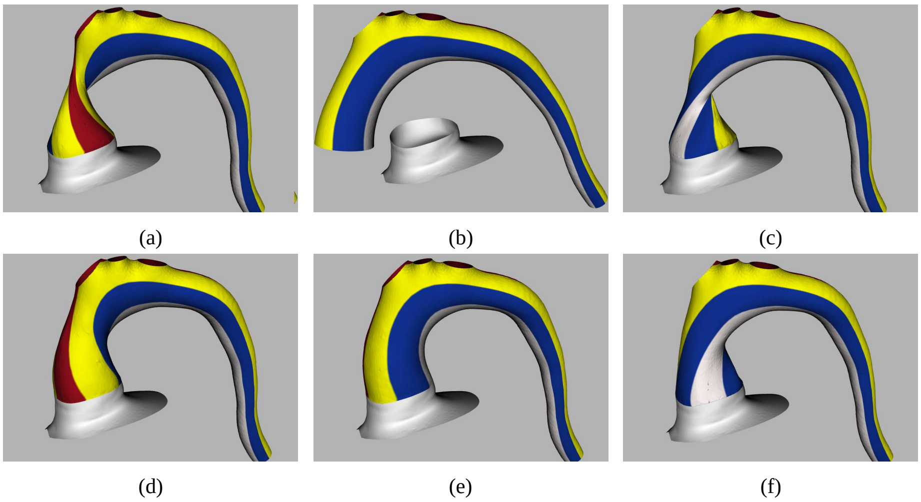


**Figure 3.** Test case 2: Joining of aorta to aortic root. Aorta is shaded in color to visualize torsion. (a, c) Large amount of torsion. (b) Initial configuration. (d, f) Medium torsion. (e) Minimum torsion configuration found by the algorithm.

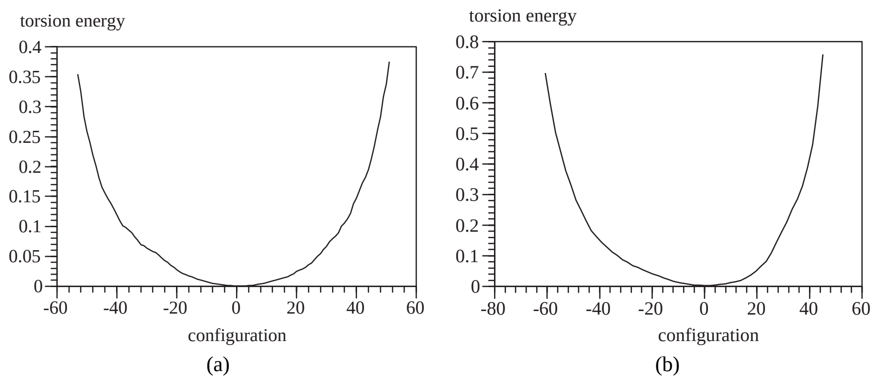


**Figure 4.** Torsion energy curves. (a) Torsion energy of test case 1. (b) Torsion energy of test case 2.

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