Introduction

- A tree share \( \tau \in T \) is inductively defined as a boolean binary tree equipped with the reduction rules \( R_1 \) and \( R_2 \) (their inverses are \( E_1 \) and \( E_2 \) respectively):

\[
\tau \xrightarrow{\text{def}} 0 \lor 1 \land \tau \lor \tau \\
R_1 : \tau \xrightarrow{\text{def}} 0 \\
R_2 : \tau \xrightarrow{\text{def}} 1
\]

(1)

- The tree domain \( T \) contains canonical trees which are irreducible with respect to the reduction rules. Here \( \circ \) denotes an “empty” leaf while \( \bullet \) a “full” leaf. The tree \( \circ \) is thus the empty tree, and \( \bullet \) the full tree. There are two “half-shares” \( \circ \circ \) and \( \bullet \circ \) and four “quarter-shares”, beginning with \( \circ \circ \).

- The domain \( T \) is equipped with the following operators:

1. \textbf{The complement} \( \bar{\tau} \).

2. The boolean function \textbf{union} \( \lor \) and \textbf{intersection} \( \land \) operator:

3. The partial \textbf{join} function \( \bullet \).

4. The \textbf{injection bowtie} function \( \Rightarrow \) generalized from string concatenation:

Properties of tree shares

- \((\land, \lor, \circ, \bullet, \circ, \bullet)\) forms a Boolean Algebra:

- \((\circ, \lor, \land, \circ, \bullet, \circ, \bullet)\) forms an Algebraic Monoid with additional properties:

Components of share solver

- \textbf{PARTITIONER}: partition the system into independent subsystems.
- \textbf{BOUNDER}: use order theory to prune space.
- \textbf{SIMPLIFIER}: apply effective heuristic heuristics for reduction the overall difficulty via computation.
- \textbf{DECOMPOSER}: decompose share system into subsystems of height zero.
- \textbf{TRANSFORMER}: for system of height zero, the component converts constants and variables from share type to boolean type.
- \textbf{INTERPRETER}: transform boolean system into equivalent boolean formula.
- \textbf{SMT_SOLVER}: check the validity of the boolean formula.
- \textbf{SAT_SOLVER}:

References