# A 3D Shape Matching Framework 

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## 1 Introduction

The research of 3D shape matching starts to get more attention recently. In this short paper, we present a 3D shape matching framework. First, we give a general definition of similarity value of two objects. Then, an algorithm is proposed consisting of Principal Component Analysis (PCA), voxelization, and iterative coarse-to-fine aligning of two objects using multi-resolution volume representation. This process will align two objects to have the maximum overlap. Then, we can compute the similarity value for shape matching. The experiment results show the effectiveness of our framework.

## 2 Similarity Value

Definition 1. The similarity value $S V$ of two 3D objects $o b j_{1}$ and $o b j_{2}$ is defined as:

$$
\begin{equation*}
S V\left(o b j_{1}, o b j_{2}\right)=\frac{\operatorname{overlap}\left(o b j_{1}, o b j_{2}\right)}{\operatorname{overall}\left(o b j_{1}, o b j_{2}\right)} . \tag{1}
\end{equation*}
$$

The above definition is consistent with our common sense: the higher the overlap-overall ratio of two objects, the higher their similarity value is. For two objects represented by volume:

Definition 2. The similarity measure $S V$ of two 3D volumes vol $l_{l}$ and $v o l_{2}$ is defined as:

$$
\begin{equation*}
S V\left(\text { vol }_{1}, \text { vol }_{2}\right)=\max _{T}\left(\frac{\| T\left(\text { vol }_{1}\right) \mathrm{I} \text { vol }_{2} \|}{\| T\left(\text { vol }_{1}\right) \mathrm{Y} \text { vol }_{2} \|}\right), \tag{2}
\end{equation*}
$$

where $\|v o l\|$ is the number of voxels in vol. The maximization max is over all possible allowed transformation $T$. In our matching scheme, they are taken from all the possible rigid object transformation such as translation, rotation, scaling and mirror imaging.

## 3 Principle Component Analysis (PCA)

By applying PCA to a 3D object, we can find the major and minor axes of the object to be matched. The object will be rotated so that the major axis is aligned with x -axis and the minor axis is aligned

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with $y$-axis. After rotation of two objects, the matching will be rotation invariant. Then, the objects will be translated so that their centers are located at the origin of the PCA coordinate system. The matching will be translation invariant. Finally, each object is scaled by a factor to normalize its length of the principle component. Now, the matching will be scaling invariant.

## 4 Voxelization

To get the volume representation of the object represented by polygon meshes, we apply voxelization. As this process mimics the scan-conversion process that pixelizes (rasterizes) 2D geometric objects, it is also referred to as 3D scan-conversion.

## 5 Iterative Coarse-to-fine Aligning of Two Objects using Multi-resolution

Using Equation 2 to compute the similarity value, an iterative process of the coarse-to-fine aligning is necessary to maximize the overlap-overall ratio. The algorithm searches the best solution in the alignment space. It is listed as follows:

```
public double similarityValue(obj1, obj2)
    // compute the initial similarity value
    sv=svPCAaligning(obj1,obj2);
    while (not the highest resolution)
        obj1=obj1.higherResolution();
        obj2=obj2.higherResolution();
        // get the next search space
        // and step for transforming objl
        space=obj1.finerSearchSpace();
        step=obj1.finerSearchStep();
        while (search not finished in space)
            obj1=obj1.transform(step);
            // method sv() uses equation 2
            if ((sv_new=obj1.sv(obj2))>sv)
                sv=sv_new;
    return sv;
```

Table 1: The algorithm of refining the similarity value: iterative coarse-to-fine aligning of two objects using multi-resolution

In the algorithm similarityValue, the similarity value $s v$ of two objects after PCA aligning is used as the initial value. The idea of the algorithm is to balance the searching speed and accuracy of the aligning operation. A coarser resolution requires less space (shorter time) to store (manipulate) the objects. When we need to search in a larger alignment space, we use lower resolution. Based on the aligning result, a smaller search space
and a higher resolution can be used in the next iteration, until the highest resolution.

## 6 Implementation and Experiment Results

We have implemented the framework and conducted a series of experiments with a database of 3D models we found from a variety of the web sites. These models comprised sets of independent polygons, without structure, adjacency information, and registered coordinate systems. The models contained polygons from 160 to 38,400 , with most of them containing around 5,000 polygons. The models used are divided into six classes, one of them is shown in Figure 1.


Figure 1: Animal class
In the following table, we list some examples of the refining similarity value of two objects using iterative coarse-to-fine aligning of multi-resolution volume representation. By this iterative process, it can improve the similarity value of using PCA only. For most of our experiments, we found that it usually converges after 3 iterations.

| Similarity Value | PCA | $1^{\text {st }}$ <br> iteration | $2^{\text {nd }}$ <br> iteration | $3^{\text {rd }}$ <br> iteration |
| :--- | :--- | :--- | :--- | :--- |
| dolphin1 vs. <br> bunny1 | 0.10 | 0.15 | 0.17 | 0.18 |
| dolphin1 vs. <br> dolphin2 | 0.67 | 0.79 | 0.85 | 0.88 |
| shark vs. whale | 0.40 | 0.45 | 0.50 | 0.53 |
| foot1 vs. foot2 | 0.52 | 0.75 | 0.80 | 0.82 |
| cow vs. lion | 0.55 | 0.66 | 0.71 | 0.73 |

Table 2: Examples of refining the similarity value: iterative coarse-to-fine aligning of two objects using multi-resolution

The similarity values of all models are illustrated in the format of bitmaps (Figure 2). Black means 1 and white means 0 . The darker the block is, the higher the similarity value of the two corresponding models is. The order of the models at x and y axes are in order of their names, i.e., AI, ant, apple, apple1, ... , whale, woman. Thus, the darkest points will be along the diagonal for the exactly same two objects. The average computing time for similarity value of two objects is about 3 min for all steps. The most time consuming steps are voxelization and iterative aligning.


Figure 2: Similarity value results
When conducting experiments on different classes, the similarity value results are shown as follows.


Figure 3: Results for different classes: (a) animals, (b) human bodies, (c) computers, (d) fruits, (e) household, and (f) transport

