This is a sample for the final exam. You can expect the exam to be of similar type of questions, but longer. No solution will be posted.

1. (a) Which of the following function is in $O(n!)$?
   (i). $2^n$  
   (ii). $n^n$  
   (iii). $(n+1)!$
   A. (i) only.  
   B. (ii) and (iii) only.  
   C. (i) and (iii) only  
   D. (i), (ii), and (iii)

   (b) What is the running time of the following method in terms of $n$?
   ```java
   int f(int n) {
     if (n == 1) return 1;
     else return f(n-1) + f(n-1);
   }
   ```
   A. $O(n)$  
   B. $O(n\log n)$  
   C. $O(n^2)$  
   D. $O(2^n)$

2. A binary search tree $T$ has the following pre-order sequence: 42137658.
   (a) Draw $T$.
   (b) Is $T$ an AVL tree?

3. You are given the text of Weiss’s "Data Structures and Problem Solving Using Java" and are asked to solve the following problem: "Give an English word w, how many times does w occur in the text?" Since the text contains many words, you want to go through the text only once, and build a suitable data structure to help you answer the question. Describe what data structure would you used, how do you build the data structure, and how you can answer the question efficiently.

4. Consider a variant of quicksort, where we partition the input arrays into three partitions instead of two: the left partition contains values less than the pivot, the middle partition contains values equal to the pivot, and the right partition contains values larger than the pivot. Describe an $O(n)$ algorithm to perform this kind of partition on an input array of integers, using the middle element as the pivot.

5. (a) Draw the unweighted graph represented by the following adjacency matrix $A$, where $A[i][j] = 1$ indicates that the edge $(i, j)$ exists.

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   (b) Give two possible outputs when we perform topological sort on the above graph.