CS3231 : Tutorial - 1

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1. Let $A = \{x, y, z\}$ and $B = \{x, y\}$.
   (a) Is A a subset of B ?
   (b) Is B a subset of A ?
   (c) What is $A \cup B$?
   (d) What is $A \cap B$?
   (e) What is $A \times B$?
   (f) What is the power set of B ?
2. If A has a elements and B has b elements, how many elements are in $A \times B$? Explain your answer.
3. If C is a set of c elements, how many elements are in the power set of C ? Explain your answer.
4. For each part, give a relation that satisfies the condition:
   (a) Reflexive and symmetric but not transitive.
   (b) Reflexive and transitive but not symmetric.
   (c) Symmetric and transitive but not reflexive.
5. Find the error in the following proof that $2 = 1$. Let $a = b = 1$, then,

$$\begin{aligned} a &= b \\ \Rightarrow a^2 &= ab \\ \Rightarrow a^2 - b^2 &= ab - b^2 \\ \Rightarrow (a+b)(a-b) &= b(a-b) \\ \Rightarrow a + b &= b \\ \Rightarrow 2 &= 1 \end{aligned}$$

6. Find the error in the following proof that all horses are the same color.
   Claim: In any set of h horses, all horses are the same color.

Proof: By induction on h.

Basis : For $h = 1$. In any set containing just one horse, all horses clearly are the same color.

Induction step: For $k \geq 1$ assume that the claim is true for $h = k$ and prove that it is true for $h = k + 1$. Take any set H of $k + 1$ horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set h_1 with just k horses. By the induction hypothesis, all the horses in H_1 are the same color. Now replace the removed horse and remove a different one to obtain the set H_2. By the same argument, all the horses in H_2 are the same color. Therefore all the horses in H must be the same color, and the proof is complete. ■

7. Show that every graph with 2 or more nodes contains two nodes that have equal degrees.