Proof of the Structure Theorem

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Structure Theorem :

There exist a $c > 0$ such that the following is true for each $\varepsilon > 0$: Every set of nodes in R^2 has a $(1+\varepsilon)$-approximate salesman path π and an associate ⅓:⅔ tiling of the bounding box such that the tour is m-light for this tiling, where $m = c \log L / \varepsilon$ and L is the size of the bounding box

Theorem 5:

Let C be any collection of straight line segments whose total length is T, and which lie entirely inside a rectangle of size W. Then the expected number of edges crossing a random line separator of the rectangle is at most 3T/W

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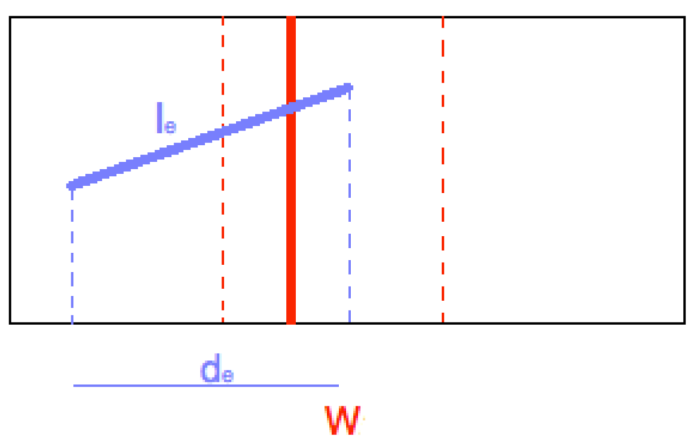

Theorem (Patching Lemma) :

There is a constant $b > 0$ such that the following is true. Let S be any line segment of length l and π be a closed path that crosses S at least thrice. Then there exist line segments on S whose total length is $b.l$ and whose addition to π changes it into a closed path that crosses S at most twice.

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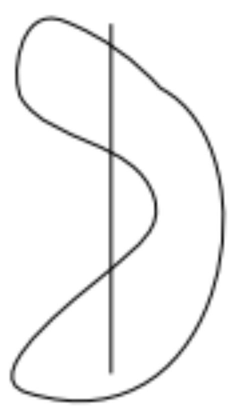

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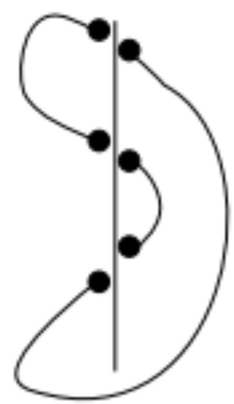

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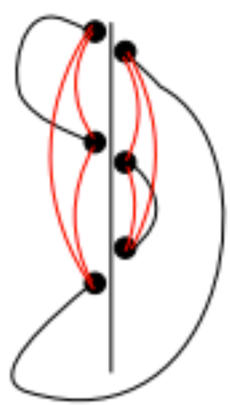

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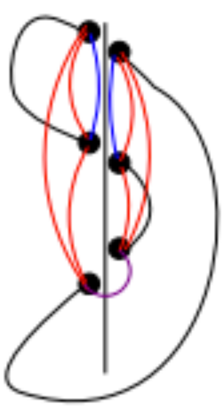

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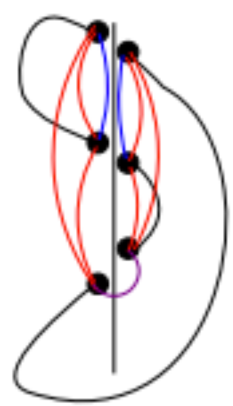

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→ an Eulerian path exists

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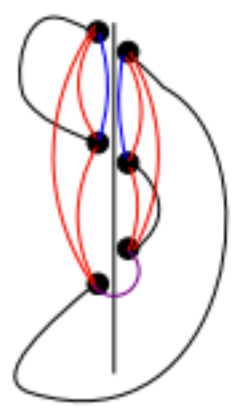

4-regular graph
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→ The theorem is proved with b = 6

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The structure Theorem : The proof

The idea is to start with an optimal tour on the original nodes and an empty tiling of the bounding box , and then to refine recursively the tiling and modify the current tour insuring it is always m-light.

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In each refining step from depth (d) to depth (d+1) :

- To partition each rectangle R with a new line separator
- To add some portals to the set of nodes
- To insure the tour crosses the line separatot at most (m) times and only at portals
- To evaluate the cost of the tour modification.

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In each refining step from depth (d) to depth (d+1) :

Two different cases :

Case 1 :

- $T \leq mW/3$

Case 2 :

- $T > mW/3$

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Case 1 :

- $T \leq mW/3$, define $\mu = 3T/W \leq m$ and apply Theorem 5
  The cost is less than $(1 + 3b/m)$

Case 2 :

- $T > mW/3$

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In each refining step from depth (d) to depth (d+1) :

Two different cases :

Case 1 :

- $T \leq mW/3$, define $\mu = 3T/W \leq m$ and apply Theorem 5
  The cost is less than $(1 + 3b/m)$

Case 2 :

- $T > mW/3$, apply the Patching Lemma with a cost $b.W \leq 3.b.T/m$
  The cost is less than $(1 + 3b/m)$

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The structure Theorem : The proof

The idea is to start with an optimal tour on the original nodes and an empty tiling of the bounding box , and then to refine recursively the tiling and modify the current tour insuring it is always m-light.

The cost is less than $(1 + 3b/m) ^ 2 \log_{1.5} L$

which is $(1 + \varepsilon)$ (when $m = 6b.\log_{1.5}L/\varepsilon$)