Geometric Algorithms
Summary of class presentation
(Group 5)

In this presentation, three geometric structures and the algorithms that efficiently construct them were introduced. More specifically, Voronoi diagrams, which can be constructed in O(n log n) time using Fortune's algorithm, Delaunay triangulation, which is a well-known problem and dual to Voronoi diagram, and trapezoidal decomposition, which can enhance point location by reducing its complexity to O(log n), were discussed. Some real-world applications for the geometric structures were also presented.

Voronoi diagram has a vast number of applications that span across several domains including astronomy, biology, cartography, marketing, meteorology, robotics and many others. (A broader taxonomy would include nearest neighbour search, largest empty circle and path planning.) The most common use of Voronoi diagram is to determine the closest site to a particular point. An example would be to answer the query “what is the closest post office with respect to my current location?”. Given a Voronoi diagram, this problem can be solved in O(log n) time. The main construction methods for Voronoi diagram are perpendicular bisector, divide-and-conquer and Fortune’s sweep line algorithm. The main concepts underlying the sweep line method were discussed. These include sweep line, beach line, site event and circle event. Moreover, the algorithm and the required data structures were also highlighted for a more comprehensive understanding of this method. The sweep line algorithm has time complexity O(n log n) and storage complexity O(n).

Delaunay triangulation is the dual graph of Voronoi diagram. It has a number of interesting properties due to the structure of Voronoi diagram, such as circumcircle property, empty circle property. Also among all triangulations, Delaunay triangulation maximizes the minimum angle. Delaunay Lema implies that we can construct a Delaunay triangulation by ensuring every edge in the triangulation is locally Delaunay. And through edge flipping we can change edges not locally Delaunay to be locally Delaunay. There are several algorithms to construct Delaunay triangulation, among which randomized incremental algorithm is a simple and fast one whose expected time complexity is O(n log n) and space complexity is O(n). Using conflict lists, we can simplify the implementation and analysis of the algorithm a lot.

Trapezoidal decomposition is mainly used to solve point location problem: given a map of areas and a point, determine in which area the point lies. One naive solution is to iterate through all areas in the map. However, if such query is frequent, then one might want to construct an efficient data structure to handle the query. One solution is build a search graph based on decomposing the map into trapezoids. Searching a point in this graph takes O(log n). We present a simple randomized incremental algorithm for trapezoidal decomposition of n segments intersecting in k points. By using backward analysis, we show that time complexity of this algorithm is O(n log n + k).

Besides these main topics, some important concepts were also introduced, such as, (1) the idea of a sweep line to solve a problem, (2) the usage of bi-directional pointers to efficiently retrieve data, (3) shear transformation to alleviate non-general position, (4) randomized incremental construction which can yield faster, and simpler to understand and implement geometric algorithms, and (5) backward analysis which can ease the analysis of some algorithms.

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