CS3230
Tutorial 8

1. Prove or disprove:

   (a) Suppose the graph contains only one edge of minimal weight. Then show that every minimal spanning tree must contain that edge.

   (b) Suppose a graph may contain several edges of minimal weight. Then does the minimal spanning tree contain all the edges of minimal weight?

2. For $n > d \geq 1$, show that $\sum_{i=d}^{n-1} \binom{i}{d} = \binom{n}{d+1}$.

3. Consider the sequence of following operations in a binomial heap starting with empty heap:

   insert 0, insert 5, insert 3, insert 6, insert 2, insert 10, insert 7, insert 8, insert 9, delete min.

   Show how the binomial heap would look after the above operations.

4. Consider the sequence of following operations in a Fibonacci heap starting with empty heap:

   insert 0, insert 1, insert 2, delete min.

   insert -1, insert -2, insert -3, delete min

   insert -4, insert -5, insert -6, delete min

   insert -7, insert -8, insert -9, delete min

   Show how the Fibonacci heap would look after the above operations

5. Suppose we use binary representation to keep a number in memory. That is, number n is represented using bits $A[k]A[k-1], \ldots A[1]$, where $A[1]$ is the least significant bit, and k is at most $1 + \log n$ (you may assume $A[k+1], A[k+2], \ldots$ are all 0).

   To increment the number, we do the following process:

Done=False

$i = 1$

While not done

If $A[i] = 0$, then set $A[i] = 1$ and done=true

Else set $A[i] = 0$

EndIf

Set $i = i + 1$.

End while

Note that if the value of the number is n, then it may take $O(\log n)$ steps to do the increment.

Show that amortized cost of increments is constant.

6. In previous question, suppose we do not start with value 0, but some arbitrary value $m \leq n$. Then, show that amortized cost of n increments is still constant, where one starts with some potential which is atmost $O(\log n)$.