CS3230
Tutorial 9

1. Consider the following problem:

   Input: Given a weighted graph G, two vertices u and v in G, and a value d.
   Question: Is there a path from u to v of weight at most d?

   Is the above problem in NP? Could it be NP-complete?

2. In class we saw that it is open at present whether $P = NP$ or not. It is also open whether $NP = EXP$ or not. Is it possible that both $P = NP$ and $NP = EXP$ are true?

3. Discrete knapsack problem is the knapsack problem we did in class where one has to either pick the whole item or none of it (i.e., we cannot pick a fraction of an item).

   It can be shown that discrete knapsack problem is NP-complete.

   Thus, if discrete knapsack problem can be solved in polynomial time, then all problems in NP can be solved in polynomial time.

   Professor S claimed that he could solve the discrete knapsack problem in time proportional to $C * n$ (see the dynamic programming algorithm done in class), where C is the capacity of the knapsack and n is the number of objects in the problem. Thus the discrete knapsack problem is in P.

   Thus, Professor S claimed that he has shown P=NP. Could you find a flaw in his argument?

4. Show that testing whether a graph $G = (V, E)$ is a subgraph of graph $G' = (V', E')$ is in NP.

5. A coloring of a graph $G = (V, E)$ is assignment of colors to each vertex of a graph such that if (u, v) is an edge, then the color assigned to u and v are different. A graph is k-colorable, if one can color the graph (with above constraint) using k colors.

   Show that checking whether a graph G is k-colorable is in NP.

6. In class you were told the decision problem regarding satisfiability (SAT).

   Consider the corresponding function problem,

   Input: A set of variables V and a set of clauses $C = \{c_1, c_2, \ldots, c_n\}$.

   Output: An assignement to the variables $v(x) \in \{true, false\}$, such that if the set of clauses is satisfiable, then the assignment $v(x)$ makes all the clauses true.

Show how you could solve the above problem, in polynomial time, if you are given a "subroutine" (as black box) to solve the SAT problem in linear time.

7. Consider the following decision problem:

   Input: A set of variables V and a boolean formula F (which uses only variables from V).

   Question: Is there a truth assignment to the variables which makes the formula true?

   (a) Is the above problem in NP?

   (b) Is the above problem NP-complete? Give reasoning for your answer.