Complexity

For definition of complexity notions, we assume the model of Turing Machine with multiple, but fixed, number of tapes. Tape alphabet is finite (fixed but can be arbitrarily large).

End markers for input: \$x\$
Can combine the end markers with the first/last symbol.

Time Complexity

$Time_M(x)$: Time/Number of steps used by a machine M on input x before halting (if M does not halt on input x, then $Time_M(x) = \infty$).

For non-deterministic Turing Machine, we assume the $Time_M(x)$ denotes the maximum time on any path, even non-accepting ones.

M is $T(n)$ time bounded, if for any input x of length n, $Time_M(x) \leq T(n)$.
We usually assume $T(n) \geq n$.

Space Complexity

Read only input tape.
$Space_M(x)$: maximum number of cells touched by M, on input x, on any of its worktapes (input tape is not counted; in many cases output tape is also not counted, which is then one-way write only tape).

If the machine does not halt on an input, then $Space_M(x)$ is taken to be infinite.

M is $S(n)$ space bounded, if for any input x of length n, $Space_M(x) \leq S(n)$.

DSPACE, DTIME, NSPACE, NTIME

$DSPACE(S(n)) = \{L \mid$ some $S(n)$ space bounded deterministic machine accepts L $\}$.

$DTIME(T(n)) = \{L \mid$ some $T(n)$ time bounded deterministic machine accepts $L\}$.

$NSPACE(S(n)) = \{L \mid$ some $S(n)$ space bounded nondeterministic machine accepts $L\}$.

$NTIME(T(n)) = \{L \mid$ some $T(n)$ time bounded nondeterministic machine accepts $L\}$.

We can similarly define the classes for function computation.