Minimization of Automata; Equivalence

Suppose we are given $A = (Q, \Sigma, \delta, q_0, F)$.

(a) We say that (p, q) are distinguishable iff there exists a string w such that either
$\hat{\delta}(p, w) \in F$, $\hat{\delta}(q, w) \notin F$, or
$\hat{\delta}(p, w) \notin F$, $\hat{\delta}(q, w) \in F$.
(b) In other words, (p, q) are indistinguisable iff for all w, $\hat{\delta}(p, w) \in F$ iff $\hat{\delta}(q, w) \in F$.

Table building algorithm for determining all pairs that are distinguishable.

1. Base Case: Initially, each pair (p, q) such that $p \in F$ and $q \notin F$, is distinguishable.

2. Inductive Step: For any $a \in \Sigma$, if $\delta(p, a)$ and $\delta(q, a)$ are distinguishable, then (p, q) are distinguishable.

3. Continue the inductive step, until it can add no more pairs of distinguishable states.

Then the remaining pairs are nondistinguishable states.

Form a new DFA as follows:

0. First delete all non-reachable states.

1. Find all nondistinguishable pairs of states.

2. Each pair of non-distinguishable states is equivalent, and it gives an equivalence relation.

3. (a) States of the new DFA are these equivalence classes.

3. (b) Transition from each equivalence class above on input a is based on the corresponding transition in original DFA, i.e., if $\delta(p, a) = q$ in the original automata, then $\delta_{new}(Ep, a) = Eq$, where Ep and Eq are equivalence classes corresponding to p and q respectively.

3. (c) Initial state of the new automata is the equivalence class containing the starting state of original automata, and final states of the new automata are all the equivalence classes containing a final state.