Q1. Basically this means that L does not have aaa as a substring.
$Q = \{q_0, q_1, q_2, q_3\}$. $\Sigma = \{a, b\}$. $F = \{q_0, q_1, q_2\}$.
$\delta(q_i, a) = q_{i+1}$, if $i \leq 2$.
$\delta(q_i, b) = q_0$, if $i \leq 2$.
$\delta(q_3, a) = \delta(q_3, b) = q_3$.
Q2. The statement is false.
Let $R = a$, $S = b$. Then, bb is in $Lang((R+\epsilon)(S+\epsilon)^*)$ but not in $Lang((R+\epsilon)^*(S+\epsilon))$.
Q3.
$S \to aaSb$
$S \to abTb \mid T \mid X$
$T \to bbTb \mid Y$
$X \to aXa \mid Y$
$Y \to bYa \mid c$
Intuitively, S first matches off the (pair of) a's to the left of c with the b's to the right of c. Then it goes to T ($abTb$) or X depending on whether the number of a's to the left of c is less or at least double the number of b's to the right of c.
T matches the (pair of) b's to the left of c with the b's to the right of c.
X matches the a's to the left of c with the a's to the right of c.
Y matches the b's to the left of c with the a's to the right of c.
They are done in appropriate order to maintain the order of a's and b's.
Q4. The grammar is ambiguous because bba has the following two different leftmost derivation.
$S \Rightarrow bbSa \Rightarrow bbRa \Rightarrow bbTa \Rightarrow bba$
$S \Rightarrow R \Rightarrow bRa \Rightarrow bTa \Rightarrow bbTa \Rightarrow bba$
Basically, the above grammar generates the language $\{b^i a^j : i \geq j\}$. The unambiguous grammar for it is:
$S \to bSa \mid T$
$T \to bT \mid \epsilon$
It is unambiguous, as any a is only generated by using the production $S \to bSa$. After using $S \to T$, only b's are generated, one by one.
Q5. True. Let $X = \{w : aba \text{ is a substring of } w\}$. Note that X is regular. Now, $Lang(G)$ is good iff $Lang(G) \cap \overline{X}$ is empty. This is used in the following algorithm to decide the question in the claim.
(0): If G is not a context free grammar, output false.
(1): Now assume G is a CFG. Convert it to a NPDA P such that $Lang(G) = Lang(P)$ as done in class.
(2): Let A be a DFA for accepting $(a+b)^*aba(a+b)^*$.
(3): Let B be a DFA for accepting the complement of $Lang(A)$.
(4): Let P' be a NPDA for accepting $Lang(P) \cap Lang(B)$ as done in class.
(5): Let G' be a grammar for $Lang(P')$ as done in class.
(6): Check if $Lang(G') = \emptyset$ as done in class.
(7): Output yes, iff the above check is true.
Q6. False.
Let $L = \{ba^iba^jba^t : M_i \text{ accepts } w_j \text{ in } \leq t \text{ time steps }\}$.
Note that L is recursive as shown in tutorials.

Now suppose $L' = substring(L)$ is recursive. Then, $L' \cap ba^*ba^*b$ must also be recursive.

But This means $\{ba^iba^jb : M_i \text{ accepts } w_j\}$ is recursive, a contradiction to result done in class.

Thus, Claim is false.

Q7.

Given M_i, design $M_{f(i)}$ such that:

$M_{f(i)}$ accepts $(ab)^j$ for all j.

Furthermore, $M_{f(i)}$ accepts aa iff $Lang(M_i)$ is not empty.

Then, $a^{f(i)} \in L_7$ iff $M_i \in L_e$ (where $L_e = \{M : Lang(M) = \emptyset\}$). Thus, $L_e \leq_m L_7$. As L_e is not r.e., we have that L_7 is not r.e.

Q8. To see that the problem is in NP, consider the following: Guess a sequence of results of the games and check if each member of the audience has their prediction true for at least one game.

To see NP-hardness, we do a reduction from 3-SAT. Suppose an instance of 3-SAT is given, which has n variables and m clauses, $C_1, C_2, \ldots, C_m$.

Then, form an instance of the problem in Q8 as follows. There are n games and m members of the audience. If $x_j \in C_i$, then i-th member of the audience predicts that A wins game j. If $\neg x_j \in C_i$, then i-th member of the audience predicts that B wins game j.

Now, if 3-SAT formula is satisfiable using truth function $t(x_j)$, then we have match fixer have A win game j iff $t(x_j)$ is true. It is easy to verify that, for $1 \leq i \leq m$, the i-th member of the audience has at least one prediction true since clause C_i is satisfied. On the other hand, if match fixer can have the results of the games such that each member of the audience has at least one prediction true, then let $t(x_j) = true$ iff A wins game j. Now, C_i is satisfied as at least one of the predictions of the i-th member of the audience is true.