Q1. Let $V = \{S, A, B, C, D, E\}$, where S is the starting symbol. The productions are as follows.
$S \to A \mid B$
$A \to CD$
$C \to aCb \mid \epsilon$
$D \to bDc \mid \epsilon$
$B \to aBc \mid E$
$E \to aEb \mid aE \mid a$
Intuitively, A generates $a^i b^j c^k$ such that $j = i+k$. For this, C generates $a^i b^i$, $i \geq 0$ and D generates $b^k c^k$, $k \geq 0$.
Intuitively, B generates $a^i b^j c^k$ such that $i > j + k$. For this, E generates $a^{j'} b^j$, $j' > j$ after B has generated $a^k E c^k$, $k \geq 0$.
Q2. $Q = \{q_0, q_1, q_2\}$, $F = \{q_2\}$, Z_0 is the starting stack symbol.
Acceptance by final state.
Transition function δ is given as follows.
$\delta(q_0, a, X) = \{(q_0, aX)\}$, for $X \in \{a, b, Z_0\}$.
$\delta(q_0, b, X) = \{(q_0, bX)\}$, for $X \in \{a, b, Z_0\}$.
$\delta(q_0, c, X) = \{(q_1, X)\}$, for $X \in \{a, b, Z_0\}$.
$\delta(q_1, \epsilon, X) = \{(q_1, \epsilon)\}$, for $X \in \{a, b\}$.
$\delta(q_1, a, a) = \{(q_1, \epsilon)\}$.
$\delta(q_1, b, b) = \{(q_1, \epsilon)\}$.
$\delta(q_1, \epsilon, X) = \{(q_2, \epsilon)\}$, for $X \in \{a, b, Z_0\}$.
Intuitively, in q_0, the NPDA pushes the input string w_1 on the stack.
On receiving c, it changes state to q_1.
In q_1, for each top of stack symbol, either it skips it (pops it out), or matches it with the input symbol.
At some point, it assumes input has finished, and transitions to q_2 (note that it needs at least one symbol on the stack for this transition).
Q3. All of the symbols are generating (B, S in the base case, then A in the inductive step). All symbols are reachable: S in base case, then A, and then B. So no uselss symbols.
B is nullable, and then A is nullable. No more nullable symbols.
Removing ϵ productions gives the grammar:
$S \to Aa \mid b \mid a$
$A \to B \mid aAB \mid aA \mid aB \mid a$
$B \to b$
The only non-trivial unit pair is (A, B). Removing it gives:
$S \to Aa \mid b \mid a$
$A \to b \mid aAB \mid aA \mid aB \mid a$
$B \to b$
Note that there are no useless symbols in above.
Converting above to CNF gives:
$S \to AC \mid b \mid a$
$A \to b \mid CD \mid CA \mid CB \mid a$
$B \to b$
$C \to a$
$D \to AB$
Q4. False.
Let $A = \{a^n b^n : n \geq 1\}$ and $B = \{c^m : m \geq 1\}$. Then, $\{xy : x \in A, y \in B, |x| = 2|y|\} = \{a^n b^n c^n : n \geq 1\}$, which was shown in class to be non-context free.
Q5. Suppose by way of contradiction that L_5 is context free. Let n be as in the pumping lemma. Without loss of generality assume $n \geq 2$.

Let $z = a^{4n}b^na^nc^n$. Note that $z \in L$ and $|z| \geq n$.

Let $z = uvwxy$ as in the pumping lemma.

Case 1: v or x contains two distinct characters.

Then, we have a subsequence ba^+b or ca^+c of uv^2wx^2y, which makes it not a member of L_5.

Case 2: Not Case 1, and vx contains b or c.

Then, if vx does not contain both b and c, then uv^2wx^2y has different number of b's and c's and thus not in L_5.

Thus, we must have $v \in b^+$ and $x \in c^+$. But then $uwy = a^{4n}b^{n-|v|}a^nc^{n-|x|}$, which is not in L_5, as $|v| > 0$. Also, here, $|vwx| > n$, which is not allowed.

Case 3: Not Case 1 and Not Case 2, and $vx \in a^+$ and vwx only contains a's from the first sequence of a's.

Then, $uwy = a^{4n-|vx|}b^na^nc^n$, which is not in L_5, as $|vx| > 0$.

Case 4: Not Case 1 and Not Case 2, and $vx \in a^+$ and vwx only contains a's from the second sequence of a's.

Then, $uwy = a^{4n}b^na^{n-|vx|}c^n$, which is not in L_5, as $|vx| > 0$.

Note that vx cannot contain a's from both first and second sequence of a's.

From the above cases, it follows that L_5 does not satisfy the pumping lemma. Thus, L_5 is not regular.