Answers to Questions in Mid Term 1

1. (a) $A = (Q, \Sigma, \delta, q_0, F)$, where
   $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$.
   $\Sigma = \{a, b\}$.
   $F = \{q_6\}$.
   δ is given by the table below.

State	a	b
q_0	q_1	q_2
q_1	q_6	q_3
q_2	q_3	q_4
q_3	q_6	q_5
q_4	q_5	q_6
q_5	q_6	q_6
q_6	q_6	q_6

   (b) $\hat{\delta}(q_0, bbab) = q_6$.

2. In the initial iteration (base case) we will note that (q_4, q_0), (q_4, q_1), (q_4, q_2), (q_4, q_3), are distinguishable pairs, as q_4 is accepting while others are not.

   In the second iteration we will note that (q_0, q_1), (q_0, q_3), (q_2, q_1), (q_2, q_3), are distinguishable as $\delta(q_3, 0) = \delta(q_1, 0) = q_4$, whereas $\delta(q_0, 0) = q_1$, and $\delta(q_2, 0) = q_3$, and (q_4, q_1) and (q_4, q_3) are distinguishable from previous iteration.

   We will not be able to add any more distinguishable pairs.

   Thus, the minimal automata would be:

   $Q = \{q_0q_2, q_1q_3, q_4\}$

   $F = \{q_4\}$.

   starting state $= q_0q_2$.

   $\delta(q_0q_2, 0) = q_1q_3$

   $\delta(q_0q_2, 1) = q_0q_2$

   $\delta(q_1q_3, 0) = q_4$

   $\delta(q_1q_3, 1) = q_0q_2$

   $\delta(q_4, 0) = q_0q_2$

   $\delta(q_4, 1) = q_1q_3$

3. (i) Construct a DFA A which accepts the same langauge as $L(S)$.

   (ii) Construct a DFA B to accept $\{w : |w| \text{ is odd}\}$.

   (iii) Construct a DFA C to accept $L(A) \cap L(B)$.

   (iv) Check if $L(C)$ is empty or not. If $L(C)$ is empty, then $L(S)$ does not contain any string of odd length. Otherwise, $L(S)$ does contain string of odd length.

4. (a) False. As a counterexample, let $L_1 = \{a^n b^n : n \geq 1\}$. Let $L_2 = \overline{L_1}$. Then both L_1 and L_2 are not regular. However, $L_1 \cap L_2 = \emptyset$ is regular.

   (b) $L((R^*S)^* + (S^*R)^*) = L(\epsilon + (R^*S)^+ + (S^*R)^+) = L(\epsilon + (R+S)^*S + (R+S)^*R) = L(\epsilon + (R+S)^+) = L((R+S)^*)$.

   (where equivalence of $(R^*S)^+$ and $(R+S)^*S$ is from Tutorial 3).

   There are several other methods to do the above.

5. Suppose by way of contradiction that L is regular. Let n be as in pumping lemma.

   Choose $m > n$ and $w = a^{8m}b^m \in L$.

   Let $w = xyz$ as in pumping lemma.

   As $|xy| \leq n$, we must have that $xy^2z = a^{8m+|y|}b^m$ must be in L. However, $8m < 8m + |y| \leq 8m + n \leq 9m < 11m$. Thus, $xy^2z \notin L$, a contradiction.

   Thus, L is not regular.