CS3231: Tutorial 1

1. Show by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

2. For a particular alphabet set Σ, how many strings of length n are there in Σ^*? How many strings in Σ^* have length $\leq n$?

3. Suppose $A, B_0, B_1, B_2, \ldots$ are languages over Σ. Show the following:

   - $A \cdot (\bigcup_{i=1}^{\infty} B_i) = \bigcup_{i=1}^{\infty}(A \cdot B_i)$.
   - $(A^*)^+ = (A^+)^* = A^*$.

4. Suppose $\Sigma = \{a, b\}$.

   (a) Let $L = \{w \mid$ number of a's in w is of form $3i + 1$, for some natural number $i\}$. Give a DFA for L.

   (b) Let $L = \{w \mid w$ has $abaab$ as a substring $\}$. Give a DFA for L.

   (c) Let $L = \{w \mid w$ does not contain $ababba$ as a substring$\}$. Give a DFA for L.

5. Given the DFA in Figure 1, determine the regular language accepted by it.
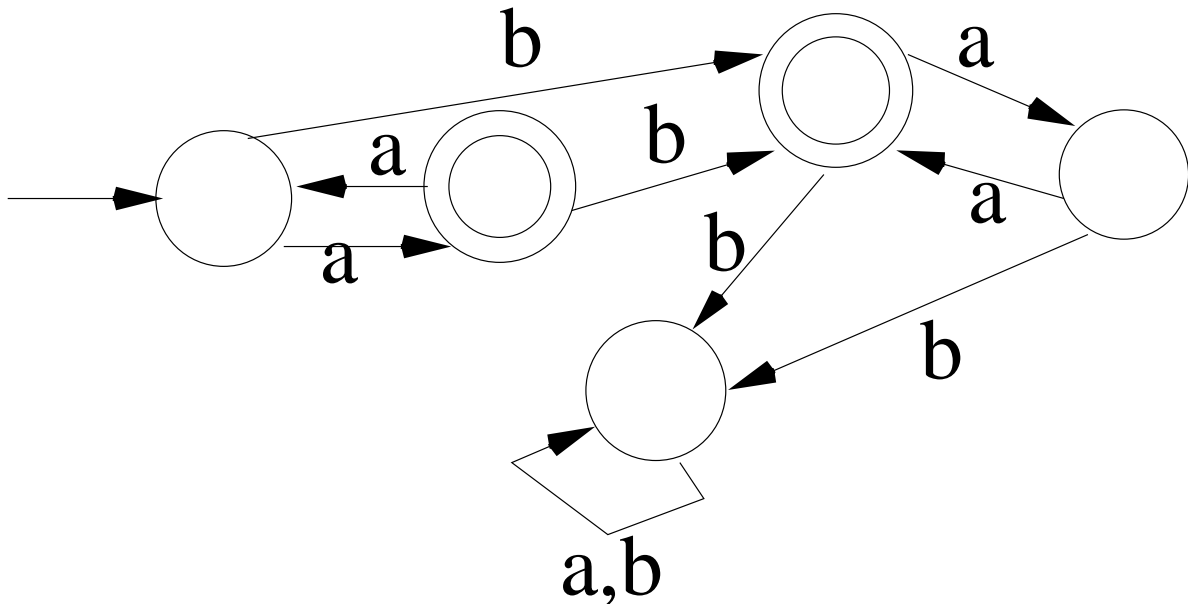

Figure 1: DFA for Q5

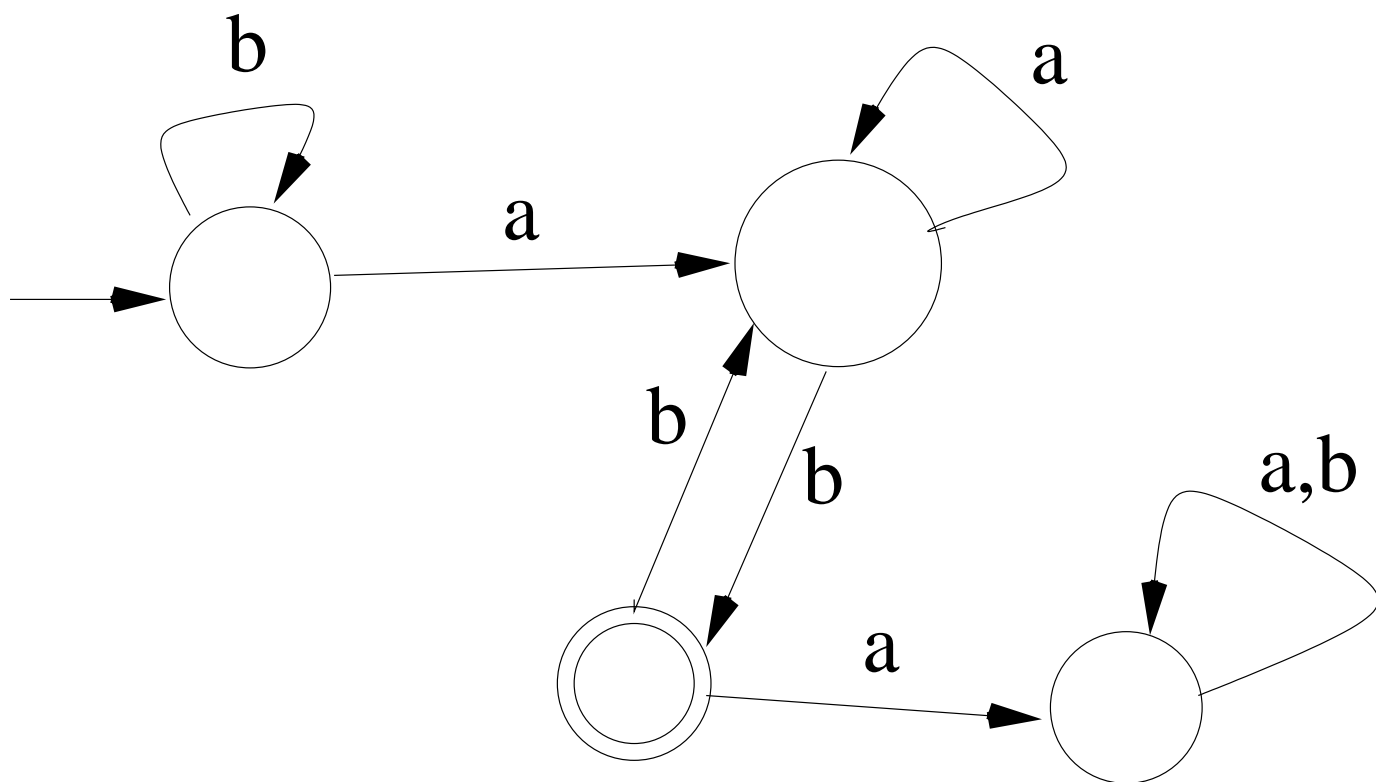6. What is the language accepted by the the DFA in Figure 2.

Figure 2: DFA for Q6