CS3231
Tutorial 10

Ans 1(a): For any machine M one can construct another machine M' such that M' accepts the same language as M and has only one accepting state, q_1 (where the starting state is q_0). Then, one can reduce L_u to the problem in Q1 by using f such that $f(M, w) = (M', q_1, w)$, where M' is obtained from M as above. Thus, the problem in the question is undecidable.

Ans 2 (a):
We reduce L_e to the language in question as follows.
Given M, $f(M) = w_i$, where i is the code for the following machine M'.
$M'(x)$
  For $t = 0$ to ∞ Do
  For $j = 0$ to t Do
    If $M(w_j)$ accepts within t steps, then accept.
  EndFor
  EndFor

Now, if $L(M) = \emptyset$, then $L(M') = \emptyset$ and thus $f(M)$ belongs to the language of the question.
If $L(M) \neq \emptyset$, then $L(M') = \Sigma^*$ and thus $f(M)$ does not belong to the language of the question.
Thus, $L_e \leq_m$ language in Q2(a).
As L_e is not RE we have that the language in Q2 (a) is not RE.

Ans 2 (b)
We reduce L_e to the language in question as follows.
Given M, $f(M) = w_i$, where i is the code for the following machine M'.
$M'(x)$
  If $x \neq b$, then reject.
  If $x = b$, then do the following.
  For $t = 0$ to ∞ Do
  For $j = 0$ to t Do
    If $M(w_j)$ accepts within t steps, then accept b.
  EndFor
  EndFor

Now, if $L(M) = \emptyset$, then $L(M') = \emptyset$ and thus $f(M)$ belongs to the language of the question.
If $L(M) \neq \emptyset$, then $L(M') = \{b\}$ and thus $f(M)$ does not belong to the language of the question.
Thus, $L_e \leq_m$ language in Q2(b).
As L_e is not RE we have that the language in Q2 (b) is not RE.

Ans 2 (c). We reduce L_e to L_{fin} as follows.

Given M, $f(M) = 1^i$, where i is the code for the following machine M'.

$M'(x)$
For $t = 0$ to ∞ Do
For $j = 0$ to t Do
If $M(w_j)$ accepts within t steps, then accept
EndFor
EndFor

Now, if $L(M) = \emptyset$, then $L(M') = \emptyset$.

If $L(M) \neq \emptyset$, then $L(M') = \Sigma^*$ (as eventually the above algorithm will find a j and $t > j$ such that $M(w_j)$ accepts within t steps).

Thus, $L_e \leq_m L_{fin}$.

Since L_e is not RE we have that L_{fin} is not RE.

Ans 2 (d). We reduce L_e to L_{inf} as follows.

Given M, $f(M) = 1^i$, where i is the code for the following machine M'.

$M'(x)$
Suppose t is such that $w_t = x$.
If there exists a $j \leq t$ such that $M(w_j)$ accepts within t steps,
then reject. Otherwise accept.
EndFor

Now, if $L(M) = \emptyset$, then $L(M') = \Sigma^*$.

If $L(M) \neq \emptyset$, then $L(M')$ is finite (to see this, note that if M accepts w_j in t' steps, then for $t \geq \max(\{j, t'\})$, M' does not accept w_t).

Thus, $L_e \leq_m L_{inf}$.

Since L_e is not RE we have that L_{inf} is not RE.

Ans 3 (a)

Decidable. Given DFAs M, M', one can algorithmically construct a DFA M'' which accepts intersection of $L(M)$ and $L(M')$ (as done in class). Given a DFA M'', one can check if $L(M'') = \emptyset$ as done in class.

Thus, one can decide if $L(M) \cap L(M') = \emptyset$.

Ans 3 (b)

Decidable.

In class we discussed an algorithm for converting a CFG G to a PDA P for accepting the same language as generated by G.

Given a PDA P and a DFA M, we discussed in class an algorithm for constructing a PDA P' for accepting the intersection of languages accepted by P and M.

Given a PDA P', we discussed in class an algorithm for constructing a CFG G' for generating the same language as accepted by P'.

Given a CFG G', we discussed in class an algorithm to determine if the language generated by G' is $\emptyset$.

Thus, using the above algorithms, we have an algorithm to determine if $L(G) \cap L(M) = \emptyset$.

Ans 3(c) Not decidable.

Let M be a DFA for accepting Σ^*. Now,

$L(M') \cap L(M) = \emptyset$, if and only if $L(M') = \emptyset$.

Thus, we can reduce L_e to language in question by having $f(M') = (M', M)$. This implies that the question is undecidable.

Ans 4: Note that membership of a machine in each of the languages in the different parts depends only on the language accepted by the machine.

Thus, by Rice's theorem, for each of the questions it is sufficient to show that the properties are nontrivial, that is, there exists a machine M which belongs to the language and there exists a machine M' which does not belong to the language.

For parts (c) and (d) we are assuming that $a \neq b$.

(a) Clearly there exists a machine which accepts $\emptyset$ and there exists a machine which accepts Σ^*. Thus, the property in part (a) is nontrivial.

(b) Clearly, there exists a machine which accepts a and a machine which does not accept a. Thus, the property in part (b) is nontrivial.

(c) Clearly, there exists a machine M which accepts a and rejects b. Furthermore, there exists a machine M' which does not accept a. Thus, the property in part (c) is nontrivial.

(d) Clearly, there exists a machine M which accepts a. Furthermore, there exists a machine M' which does not accept a and accepts b. Thus, the property in part (d) is nontrivial.

Ans 5: False.

For this, consider $L = L_u \cup \Sigma$. Note that L is not recursive, as L and L_u differ only on a finite set (see Tutorial 9).

However, $L^* = \Sigma^*$. Thus, L^* is recursive.