CS3231
Tutorial 11

1. Use the construction of Universal Turing Machine to show that the following language is recursive:

   $\{(a^i, a^j, a^t) : M_i \text{ accepts } w_j \text{ in } \leq t \text{ time steps } \}$

   Here M_i is the i-th Turing Machine, w_j is the j-th string and t is a natural number.

2. In the construction of Universal Turing Machine done in class, suppose we allow the coded Turing Machine to be non-deterministic. Then give a construction of a non-deterministic Universal Turing Machine which takes two inputs: p, a code for non-detereministic Turing Machine, and x; and then simulates the Turing Machine with code p on input x (for language acceptance).

   You need not describe the whole construction, but just give the main points about where the above Universal Turing Machine differs from the Universal Turing Machine for deterministic TMs.

3. Suppse L_1, L_2 are recursively enumerable sets.

   (a) Is $L_1 \cap L_2$ recursively enumerable?

   (b) Is $L_1 - L_2$ recursively enumerable?

4. The state entry problem for Turing Machines can be described as follows. Suppose a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$, a state $q \in Q$, and input $w \in \Sigma^*$ is given. Does M ever enter the state q when input is string w?

   Show that above problem is undecidable using appropriate reduction from the Halting problem.

5. Recall that W_i denotes the language accepted by the Turing Machine with code i. That is, $W_i = L(M_i)$.

   (a) Show that $L_{inf} = \{1^i \mid W_i \text{ is infinite } \}$ is not RE.

   (b) Show that $L_{fin} = \{1^i \mid W_i \text{ is finite } \}$ is not RE.

   (c) Show that $\{w_i \mid a \in W_i \text{ or } b \notin W_i\}$ is not RE, where $a \neq b$.

   (d) Show that $L_a = \{w_i \mid a \notin W_i\}$ is not RE.

   Note that Rice's theorem cannot be used to show a language over machines is not RE. So you would need to use some of the techniques used in class.

6. Which of the following problems are decidable and why?

   (a) Given two DFA's M_1 and M_2, is $L(M_1) \cap L(M_2) = \emptyset$?

   (b) Given a CFG G, and a DFA M, is $L(G) \cap L(M) = \emptyset$?

   (c) Given a TM M', and a DFA M, is $L(M') \cap L(M) = \emptyset$?

7. Show that the following languages are undecidable, using Rice's theorem.

   (a) $\{M \mid \text{number of elements in } L(M) \leq 20\}$.

   (b) $\{M \mid a \in L(M)\}$.

   (c) $\{M \mid a \in L(M) \text{ and } b \notin L(M)\}$.

   (d) $\{M \mid a \in L(M) \text{ or } b \notin L(M)\}$.