CS3231
Tutorial 12

1. Show that $DSPACE(n^2)$ is a proper subset of $DSPACE(n^3)$.

2. State whether the following are true or false, and give a short justification for your answer.

   (a) For every context free language A there exists a regular language B such that $A \leq_m B$.

   (b) Given a simple graph $G = (V, E)$, with n vertices, it is decidable in polynomial time whether G can be colored using $n - 1$ colors. That is, it is decidable in polynomial time, whether there is a mapping f from V to $\{1, , 2, \ldots, n - 1\}$ such that for all $(u, v) \in E$, $f(u) \neq f(v)$.

   For the following problems you may assume that Hamiltonian Circuit and Partition are NP-complete.

3. Show that Travelling Salesman problem is NP-complete.

4. Consider the following problem:

   INSTANCE: A graph $G = (V, E)$ and an integer k.

   QUESTION: Is there a simple path in G of length at least k?

   Show that the above problem is NP-complete.

5. Consider the following problem:

   INSTANCE: A number k of processors, and a set $J = \{J_1, J_2, \ldots, J_r\}$ of jobs where job J_i has running time T_i, and a deadline D.

   QUESTION: Is there a way to schedule the jobs on the k-processors (non-preemptively, i.e., a job has to run on the processor it is allocated to until completion) such that the total time taken to finish all the jobs is at most D?

   Show that the above problem is NP-complete.

6. Savitch showed that if $S(n) \geq \log n$, and $S(n))$ is fully space constructible, then

   $$NSPACE(S(n)) \subseteq DSPACE((S(n))^2).$$

   Let PSPACE denote the class of languages which can be accepted in polynomial space (in the length of the input) by a deterministic multi-tape Turing machine. Let NPSPACE denote the class of languages which can be accepted in polynomial space (in the length of the input) by a non-deterministic multi-tape Turing Machine.

   Show that PSPACE=NPSPACE.

   Note: Don't worry about "fully space constructible" requirement above. That is some technical requirement, which just says that the functions used are "nice" in some way. Polynomial functions such as n^3, n^{56} etc are fully space constructible (nice).