Tutorial 2

1. Let $A = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{0, 1\}, \delta, q_0, \{q_3\})$, where

   $\delta(q_0, 0) = q_2$

   $\delta(q_2, 0) = q_0$

   $\delta(q_2, 1) = \delta(q_5, 0) = \delta(q_5, 1) = q_5$

   $\delta(q_0, 1) = q_1$

   $\delta(q_1, 1) = q_0$

   $\delta(q_1, 0) = q_3$

   $\delta(q_3, 0) = \delta(q_3, 1) = q_4$

   $\delta(q_4, 0) = \delta(q_4, 1) = q_3$

2. By induction on $|y|$. Clearly, for $y = \epsilon$, the statement holds.

   Suppose the statement holds for $y = w$. Then for $y = wa$, with $a \in \Sigma$, we have

   $\hat{\delta}(q, xwa) = \delta(\hat{\delta}(q, xw), a) = \delta(\hat{\delta}(\hat{\delta}(q, x), w), a) = \hat{\delta}(\hat{\delta}(q, x), wa)$.

3. $A = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_3\})$, where

   $\delta(q_0, a) = \{q_0\}$

   $\delta(q_0, b) = \{q_0, q_1\}$

   $\delta(q_1, b) = \{q_2\}$

   $\delta(q_2, a) = \{q_3\}$

   (rest of the transitions are $\emptyset$).

   DFA equivalent to it has the starting state $\{q_0\}$. The transition table is given in Table 1.
   The accepting states are: $\{q_3\}, \{q_1, q_3\}, \{q_0, q_3\}, \{q_2, q_3\}, \{q_0, q_1, q_3\}, \{q_0, q_2, q_3\}, \{q_1, q_2, q_3\}, \{q_0, q_1, q_2, q_3\}$.

	a	b
$\emptyset$	$\emptyset$	$\emptyset$
$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_1\}$	$\emptyset$	$\{q_2\}$
$\{q_2\}$	$\{q_3\}$	$\emptyset$
$\{q_3\}$	$\emptyset$	$\emptyset$
$\{q_0, q_1\}$	$\{q_0\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_3\}$	$\{q_0, q_1\}$
$\{q_0, q_3\}$	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_1, q_2\}$	$\{q_3\}$	$\{q_2\}$
$\{q_1, q_3\}$	$\emptyset$	$\{q_2\}$
$\{q_2, q_3\}$	$\{q_3\}$	$\emptyset$
$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_3\}$	$\{q_0\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_3\}$	$\{q_0, q_1\}$
$\{q_1, q_2, q_3\}$	$\{q_3\}$	$\{q_2\}$
$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_2\}$

Table 1: Transition Table for Q3

4. (a) q_0 is starting state and q_2 is accepting state.

	ϵ	a	b
q_0	$\{q_1\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_0\}$	$\{q_1\}$	$\{q_2\}$
q_2	$\{q_1\}$	$\{q_1\}$	$\{q_2\}$

(b) $Eclose(q_0) = \{q_0, q_1\}$

$Eclose(q_1) = \{q_0, q_1\}$

$Eclose(q_2) = \{q_0, q_1, q_2\}$

(c)

$\hat{\delta}(q_2, a) = \{q_0, q_1\}$

$\hat{\delta}(q_2, b) = \{q_0, q_1, q_2\}$.

(d) The automata accepts any string ending in b.

$A = (\{q_0, q_1\}, \{a, b\}, \delta, q_0, \{q_1\})$

$\delta(q_0, a) = \delta(q_1, a) = q_0$.

$\delta(q_0, b) = \delta(q_1, b) = q_1$.

5. (a) $L((R + S)^*) = L((R^* S^*)^*)$, for any regular expressions R and S.

Ans: True:

Showing $\subseteq$:

$L(R) \subseteq L(R^*S^*)$ and $L(S) \subseteq L(R^*S^*)$. Thus $L(R+S) \subseteq L(R^*S^*)$. Thus, $L((R+S)^*) \subseteq L((R^*S^*)^*)$.

Showing $\supseteq$:

$L(R^*S^*) \subseteq L((R+S)^*(R+S)^*) = L((R+S)^*)$

Thus, $L((R^*S^*)^*) \subseteq L(((R+S)^*)^*) = L((R+S)^*)$ (As $L(A^*) = L((A^*)^*)$.

(b) False. Take $\Sigma = \{a, b\}$ with $a \neq b$. Let $S = a$, $R = b$. Then, $ababa$ is in $L(S(R+S)^*S)$ but not in $L((SR^*S)^+)$.

6. $R^0_{11} = 1 + \epsilon$

   $R^0_{12} = 0$

   $R^0_{21} = 0$

   $R^0_{22} = 1 + \epsilon$

   $R^1_{12} = R^0_{12} + R^0_{11}(R^0_{11})^* R^0_{12} = 0 + (1+\epsilon)(1+\epsilon)^*0 = 1^*0$

   $R^1_{22} = R^0_{22} + R^0_{21}(R^0_{11})^* R^0_{12} = (1+\epsilon) + 0(1+\epsilon)^*0 = 1 + \epsilon + 01^*0$

   $R^2_{12} = R^1_{12} + R^1_{12}(R^1_{22})^* R^1_{22} = 1^*0 + 1^*0(1+\epsilon+01^*0)^*(1+\epsilon+01^*0) = 1^*0 + 1^*0(1+\epsilon+01^*0)^* = 1^*0 + 1^*0(1+01^*0)^* = 1^*0(1+01^*0)^*$

   The language accepted by the NFA is R^2_{12}.