CS3231
Tutorial 3

Notation: $\overline{L}$ denotes the complement of L, that is, $\overline{L} = \Sigma^* - L$.

1. Consider the DFA given in Figure 1. Give the minimal DFA which accepts the same language as accepted by the DFA in figure 1.

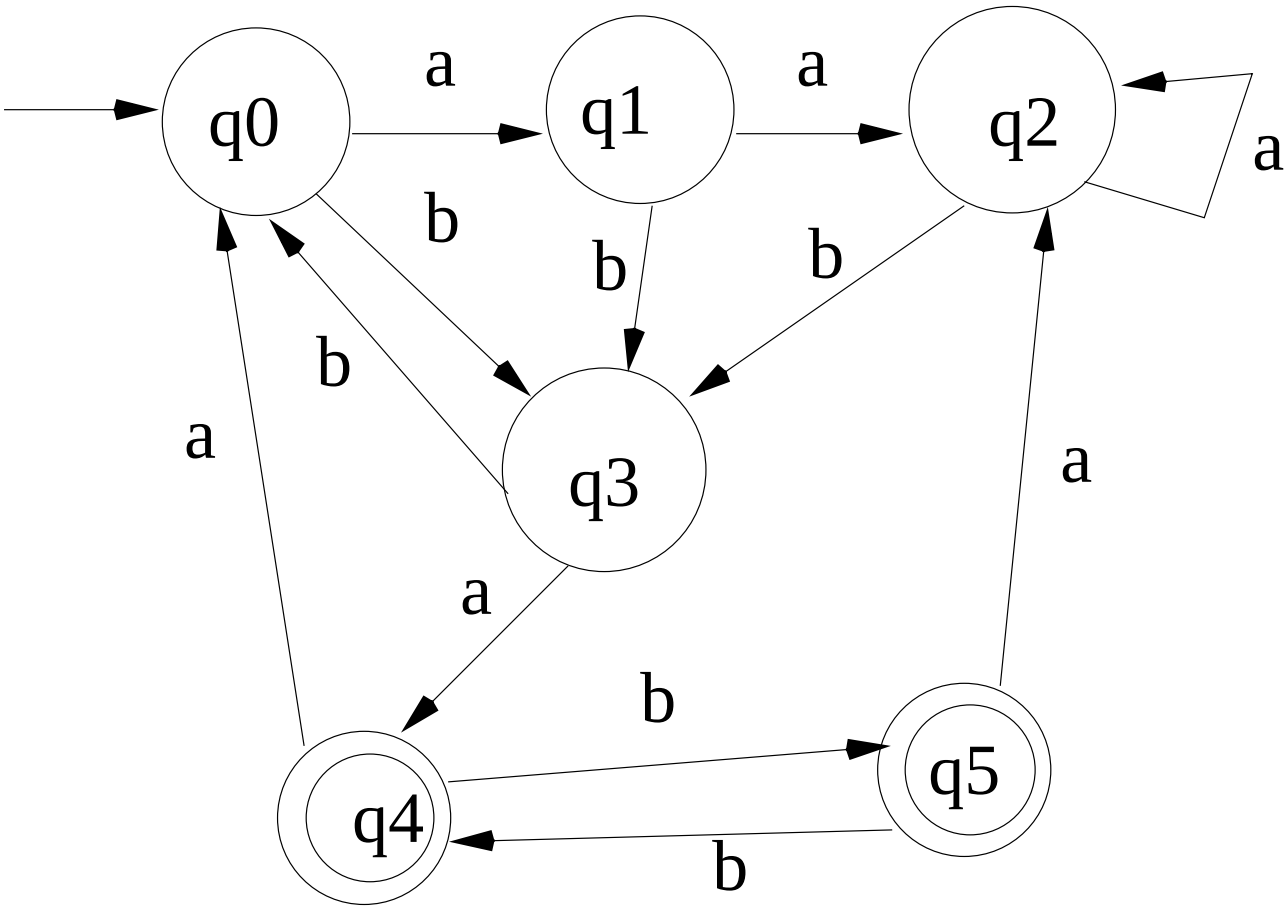

Figure 1: DFA for Q1

2. Prove or disprove the following statements.

(a) Suppose A and B are regular. Then $A \cdot \overline{B}$ is also regular.

(b) Suppose L is a regular language. Then, $L^R = \{x^R \mid x \in L\}$ is also a regular language. Here x^R denotes the reverse of a string x.

(c) If L_1 is regular and $L_2 \subseteq L_1$, then L_2 is regular.

(d) If L_1 is regular and $L_1 \cup L_2$ is regular, then L_2 is regular.

(e) Suppose that L_1 and L_2 are regular languages. Then, $L_1 \cap L_2$ is also a regular language.

(f) For a string x, let x_i denote the i-th character in x. That is, $x = x_1x_2x_3 \ldots x_n$, where n is the length of x and each $x_i \in \Sigma$.
Suppose L is regular. Then $\{x :$ for some natural number r, $|x| = 2r$ and $x_1x_3x_5 \ldots x_{2r-1} \in L\}$ is also regular.

3. Which of the following languages are regular? Prove your answer. Below w^R denotes the reverse of w.

(a) $\{wcw^R \mid w \in \{a, b\}^*\}$ (where $\Sigma = \{a, b, c\}$).

(b) $\{ww \mid w \in \{a, b\}^*\}$.

(c) $\{wxw^R \mid w, x \in \{a, b\}^+\}$.

(d) $L = \{a^m : m > 0$ and binary representation of m has even number of bits $\}$. Here binary representation of a number > 0 starts with a 1. Thus, representation of 5 is 101 and not 00101.

4. Consider $L = \{b^m \mid m \geq 0\} \cup \{a^m b^p \mid m \geq 1, p$ is prime number$\}$. Show that L satisfies the pumping lemma. (However, L is not a regular language).