CS3231
Tutorial 4

1. (Hard) For any language L, let $HALF(L) = \{w : (\exists u)[wu \in L \text{ and } |w| = |u|]\}$.

   Show that if L is regular, then $HALF(L)$ is regular.

2. Give an ϵ-NFA for the language generated by the following right-linear grammar.

$$S \to abA|aaB|\epsilon$$
$$A \to baA|bB$$
$$B \to aS$$

3. The right-linear grammars we studied in class have productions of the form: $V \to T^*(V \cup \{\epsilon\})$ (that is, the non-terminal on the RHS, if any, is at the right end). A left-linear grammar is one in which the productions are of the form: $V \to (V \cup \{\epsilon\})T^*$ (that is, the non-terminal on the RHS, if any, is at the left end).

   Recall that x^R denotes the reverse of string x.

   (a) Let $L^R = \{w^R : w \in L\}$. It was earlier shown that if L is regular then so is L^R.

   (b.1) Suppose G is a right-linear grammar for L. Show how to produce a left-linear grammar for L^R, using G.

   (b.2) Suppose G is a left-linear grammar for L. Show how to produce a right-linear grammar for L^R, using G.

   (c) Using (a) and (b) show that left-linear grammars generate exactly the regular languages.

4. Give context free grammars for the following languages over the alphabet Σ:

   (a) $L = \{cwcw^Rc : w \in \{a, b\}^*\}$. $\Sigma = \{a, b, c\}$.
   (b) $L = \{a^m b^n : 2m \geq n\}$. $\Sigma = \{a, b\}$.
   (c) $L = \{w : \text{number of } a\text{'s in } w \text{ is the same as the number of } b\text{'s in } w\}$. $\Sigma = \{a, b\}$.

5. Consider the grammar given in the previous question for $L = \{w : \text{number of } a\text{'s in } w \text{ is the same as the number of } b\text{'s in } w\}$.

   Give a parse tree for $abbaab$.

6. The context free grammar:

   $S \rightarrow aSb|aSa|bSa|bSb|\epsilon$

   is neither a right-linear nor a left-linear grammar. However the language generated by above grammar is regular. Determine the language, and give a right-linear grammar for the language.

7. (a) Show that the following grammar is ambiguous:

$$S \rightarrow bA|aB$$
$$A \rightarrow a|aS|bAA$$
$$B \rightarrow b|bS|aBB$$

   (b) Find unambiguous grammar for the language generated by the grammar in part (a).