

CS3231

Tutorial 4

1. Which of the following languages are regular? Prove your answer. Below w^R denotes the reverse of w .

- (a) $\{a^n b a^m \mid m \leq n\}$.
- (b) $\{w c w^R \mid w \in \{a, b\}^*\}$.
- (c) $\{w w \mid w \in \{a, b\}^*\}$.
- (d) $\{w x w^R \mid w, x \in \{a, b\}^+\}$.

2. Show that $L = \{a^m \mid m > 0 \text{ and decimal representation of number } m \text{ has odd number of digits}\}$ is not regular.

Note: Decimal representation of a (non-zero positive) number starts with a non-zero digit. For example decimal representation of number twenty-two is 22 and not 022, 0022 etc. Thus L contains strings $a, a^2, a^3, \dots, a^9, a^{100}, a^{101}, \dots, a^{999}, a^{10000}, \dots$

Hint: Note that gap between $10^r - 1$ and 10^{r+1} becomes large as r increases.

3. Use results done in class to show that $\{w \mid \text{number of } a\text{'s in } w \text{ is same as number of } b\text{'s in } w\}$ is not regular.
4. Consider $L = \{b^m \mid m \geq 0\} \cup \{a^m b^p \mid m \geq 1, p \text{ is prime number}\}$. Show that L satisfies the pumping lemma. (However, L is not a regular language).
5. (Hard) For any language L , let $HALF(L) = \{w \mid (\exists u)[w u \in L \text{ and } |w| = |u|]\}$. Show that if L is regular, then $HALF(L)$ is regular.