# CS3231
# Tutorial 5

1. Give a $\epsilon$-NFA for the language generated by the following right-linear grammar.

$$S \to abA|aaB|\epsilon$$
$$A \to bbA|bB$$
$$B \to bS$$

2. The right-linear grammars we studied in class have productions of the form: $V \to T^*(V \cup \{\epsilon\})$ (that is, the non-terminal on the RHS, if any, is at the right end). A left-linear grammar is one in which the productions are of the form: $V \to (V \cup \{\epsilon\})T^*$ (that is, the non-terminal on the RHS, if any, is at the left end).

   (a) Let $L^R = \{w^R \mid w \in L\}$. We showed in class that if $L$ is regular then so is $L^R$.

   (b.1) Suppose $G$ is a right-linear grammar for $L$. Show how to produce a left-linear grammar for $L^R$, using $G$.

   (b.2) Suppose $G$ is a left-linear grammar for $L$. Show how to produce a right-linear grammar for $L^R$, using $G$.

   (c) Show using (a) and (b) that left-linear grammars generate exactly the regular languages.

3. Give context free grammars for the following languages:

   (a) $L = \{cwcw^Rc \mid w \in \{a, b\}^*\}$.
   (b) $L = \{a^m b^n \mid m \leq n\}$.
   (c) $L = \{w \mid$ number of $a$'s in $w$ is same as number of $b$'s in $w\}$.

4. Consider the grammar given in the previous question for $L = \{w \mid$ number of $a$'s in $w$ is same as number of $b$'s in $w\}$.

   Give a derivation tree for $abbaab$.

5. The context free grammar:

   $S \to aSb|aSa|bSa|bSb|\epsilon$

   is not a right-linear (or left-linear) grammar. However the language generated by above grammar is regular. Determine the language, and give a right-linear grammar for the language.

6. (a) Show that the following grammar is ambiguous:

$$S \to bA|aB$$
$$A \to a|aS|bAA$$
$$B \to b|bS|aBB$$

   (b) Find unambiguous grammar for the language generated by the grammar in part (a).