Tutorial 7

1. Use dynamic programming algorithm to determine whether $aabaabba$ belongs to the language generated by the grammar:

$$S \to AB$$
$$B \to b|AC$$
$$A \to CD$$
$$C \to a|b|DA$$
$$D \to a|b$$

2. Prove or disprove that the following are context-free languages. Below $\Sigma = \{a, b, c, d\}$.

   (a) $\{a^i b^j : i = j^2\}$.
   (b) $\{a^n b^m c^q \mid n \neq m \text{ or } m \neq q\}$.
   (c) $\{a^n \mid n \text{ is a prime }\}$.
   (d) $\{a^i b^j c^k d^l \mid i = 0 \text{ or } j = k = l\}$.
   (e) $\{udv : u, v \in \{a, b, c\}^* \text{ and } u \text{ is a substring of } v\}$.

3. For a language L, let $Prefix(L) = \{x : (\exists y)[xy \in L]\}$.

   (a) Prove or Disprove: If L is context free, then $Prefix(L)$ is also context free.

   (b) Prove or disprove: If L is a regular language, then $Prefix(L)$ is also a regular language.

4. There is a stronger version of pumping lemma known as Ogden's lemma, given below. Prove it. (Method of proof is essentially similar to pumping lemma, except that we concentrate on distinguished positions, rather than any symbol in z).

   Ogden's Lemma: Let L be a CFL. Then there exists a constant n such that the following holds for any string z of length at least n in L. If we mark at least n positions in z to be distinguished, then we can write $z = uvwxy$ such that:

   (a) vwx has at most n distinguished positions;

   (b) vx has at least one distinguished position;

   (c) For all i, $uv^i wx^i y$ is in L.

5. Use Ogden's Lemma to show that

   $\{0^r 1^s 2^t \mid r = s \text{ and } s \neq t\}$ is not context free.

   Hint: Take n as in Ogden's Lemma. Then use a string of the form $0^n 1^n 2^t$, for some appropriate value of t, and mark the 0's as distinguished.