CS5230:Tutorial 1

1. Consider the problem of sorting m numbers that you studied in your algorithms class.

   (a) Model how input for above problem can be given to the Turing Machine.

   (b) Consider any algorithm for sorting (you may want to choose a simple algorithm). Show how you might implement it on the Turing Machine. You may just give a sketch of how the Turing Machine does important steps of the algorithm. You need not give the state table of Turing Machine itself.

   (c) How much time does your algorithm take? Giving the time complexity in O notation is fine.

2. (a) Give details of how one can simulate a multi-tape TM using one tape TM.

   (b) Consider the simulation of multi-tape Turing Machine done in part (a). If the multi-tape TM takes time t before it halts on input x, can you provide a reasonable bound on the time taken by the simulation?

   For the following questions, n is the length of the input.

3. (a) Suppose a Turing Machine M is n-time bounded. Can we construct another Turing Machine which accepts the same language as M, but is $n/2$ time bounded?

   (b) Suppose a Turing Machine M is n-space bounded. Can we construct another Turing Machine which accepts the same language as M, but is $n/2$ space bounded?

4. Can you give a recursive language, which cannot be accepted in time $O(2^n)$? $O(2^{2^{2^{2^n}}})$?

5. a) Show that n^2, 2^n, $n\lceil \log n \rceil$, are time constructible functions.

   b) Show that n, $n\lceil \log n \rceil$, $\lceil \log n \rceil$, are space constructible functions.

6. Assume $T(n)$ is super linear (that is $\lim_{n\to\infty} \frac{T(n)}{n} = \infty$).

   Show that if $T(n)$ can be computed, in binary, within $T(n)$ time steps, then $T(n)$ is fully time constructible.