Tutorial 10:

Q1: Show that if $L_1 \in PP$, $L_2 \in PP$, then $(L_1 - L_2) \cup (L_2 - L_1) \in PP$.

(* $(L_1 - L_2) \cup (L_2 - L_1)$ is also called symmetric difference of L_1 and L_2. *)

Q2: Show that QBF is PSPACE-complete.

Q3: A set L is said to be self-reducible iff there exists a polynomial time computable function f such that, for all x,

(a) $f(x) = 0$ or $f(x) = 1$ or $f(x) \subseteq \Sigma^*$.

(b) if $f(x) = 0$ then $x \notin L$;

(c) if $f(x) = 1$ then $x \in L$;

(d) if $f(x) \subseteq \Sigma^*$ then $x \in L$ iff at least one of member of $f(x)$ is in L. Furthermore, each member of $f(x)$ is of length strictly smaller than the length of x.

Show that SAT is self-reducible.

Q4. Suppose we additionally require in Q3 that in part (d), $f(x)$ consists of exactly one element (rather than a set of elements). Then show that if L is self-reducible (according to new definition), then L is in P.

Q5: Consider the interactive proof protocol studied in class. Let α, β be rational numbers.

Define $IP_{\alpha,\beta}$ as the complexity class consisting of language L for which there exists a polynomial time bounded randomized verifier V and a prover P (as in the definition of IP done in the class), such that:

(1) If $x \in L$, then after the interaction of P and V, V accepts with probability $\geq \alpha$;

(2) If $x \notin L$, then for any prover P', after the interaction of P' and V, V accepts with probability $\leq \beta$.

(a) Show that $\mathbf{NP} \subseteq IP_{1,0}$.

(b) Show that $IP_{2/3,0} \subseteq \mathbf{NP}$.