Tutorial 5 (and 6)

Show that the following problems are NP-complete.

(a) Vertex Cover

Input: a (undirected) graph $(V, E)$, and a number $k$.

Question: Is there a subset $V'$ of $V$ such that $V'$ has at most $k$ elements, and for every edge $(u, v)$ in $E$, at least one of $u, v$ belongs to $V'$?

(b) Clique

Input: a (undirected) graph $(V, E)$ and a number $k$.

Question: Is there a subset $V'$ of $V$ of size $k$ such that for each distinct $u, v$ in $V'$, $(u, v)$ is in $E$?

(c) X3C

Input: A finite set $A$, with $|A| = 3q$, and a collection $C$ of 3-element subsets of $A$.

Question: Does $C$ contain an exact cover for $A$, that is, a subcollection $C' \subseteq C$ such that every element of $A$ occurs in exactly one member of $C'$.

(d) 3-Colorability.

Input: An undirected graph $G = (V, E)$.

Question: Is there a mapping $\mathit{color} : V \to \{1, 2, 3\}$ such that for all $(v, w) \in E$, $\mathit{color}(v) \neq \mathit{color}(w)$?

(e) Not-All-Equal SAT (NAESAT).

Input: A set of variables $V$, and a set $C$ of clauses, each having exactly three literals.

Question: Is there a truth assignment to the variables so that each clause has at least one true literal and at least one false literal?

(f) MAX2SAT

Input: A set of variables $V$, a number $k$ and a set of clauses $C$, where each clause has at most two literals.

Question: Is there a truth assignment to the variables such that at least $k$ of the clauses become true?

(g) Hamiltonian Circuit

Input: A graph $G (= (V, E)$

Output: Does $G$ contain a Hamiltonian Circuit, that is, an ordering $(v_1, \ldots, v_n)$ of the vertices in $V$ such that $(v_i, v_{i+1}) \in E$, for $1 \leq i < n$, and $(v_n, v_1) \in E$.