Tutorial 8:

For the following questions:

A class $\mathcal{L}$ being closed under intersection means that, for all $L_1, L_2 \in \mathcal{L}$, $L_1 \cap L_2 \in \mathcal{L}$. Similar definitions hold for closure under other operators such as union, complementation etc.

Q1: Show that NP is closed under union and intersection.

Q2: Show that BPP is closed under union, intersection, and complementation.

Q3: Suppose NP is closed under complementation. Then show that, for any $L \in NP$, there exists a nondeterministic polytime bounded Turing machine $\mathbf{M}$ such that (a) if $x \in L$, then $M(x)$ accepts on some path, and rejects on no path. (b) if $x \notin L$, then $M(x)$ rejects on some path, and accepts on no path. The machine M above may neither accept, nor reject on some of the paths.

Q4. Show that $ZPP = R \cap coR$.

Q5. Consider the following variation of the definition of PP:

L is in PP' iff there exists a polynomial time bounded probabilistic turing machine M such that $x \in L$ iff $Prob_M(x) > 1/2$. (i.e, if the probability of acceptance is exactly half, then we reject the input)

Show that $PP = PP'$.