Tutorial 9:

Q1: (a) Show that $BPP^{BPP} = BPP$.

(b) Show that $PSPACE^{PSPACE} = PSPACE$.

Q2. Consider the following variation of the class PP. We define the class PP'' as follows. L is in PP'', iff there exists a polynomial time bounded probabilistic turing machine M such that, $x \in L$ iff $Prob_M(x) > 1/4$.

Show that $PP = PP''$.

Q3: A set L is said to be self-reducible iff there exists a polynomial time computable function f such that, for all x,

(a) $f(x) = 0$ or $f(x) = 1$ or $f(x) \subseteq \Sigma^*$.

(b) if $f(x) = 0$ then $x \notin L$;

(c) if $f(x) = 1$ then $x \in L$;

(d) if $f(x) \subseteq \Sigma^*$ then $x \in L$ iff at least one of member of $f(x)$ is in L. Furthermore, each member of $f(x)$ is of length strictly smaller than the length of x.

Show that SAT is self-reducible.

Q4. Suppose we additionally require in Q3 that in part (d), $f(x)$ consists of exactly one element (rather than a set of elements). Then show that if L is self-reducible (according to new definition), then L is in P.

Q5: Recall that $PSPACE = \bigcup_i DSPACE(n^i)$

Show that if $PSPACE \subseteq P/poly$, then $PSPACE \subseteq \Sigma_2^p$.