Translation Lemma Example

$DTIME(2^n)$ is a proper subset of $DTIME(n2^n)$.

Proof: It is easy to show that 2^n and $n2^n$ are fully time constructible.

Note that time hierarchy theorem cannot be directly used as $\lim_{n \to \infty} 2^n(log(2^n))/(n2^n) = 1$.

Suppose by way of contradiction that $DTIME(n2^n) \subseteq DTIME(2^n)$.

Take $S_1(n) = n2^n$, $S_2(n) = 2^n$ and $f_1(n) = 2^n$.

Then, we have by translation lemma:

(A) $DTIME(2^n 2^{2^n}) \subseteq DTIME(2^{2^n})$.

If we use $f_2(n) = n + 2^n$, then we get Then, we have by translation lemma:

(B) $DTIME((n + 2^n)2^{n+2^n}) \subseteq DTIME(2^{n+2^n})$.

Combining, (A) and (B), we get:

(C) $DTIME((n+2^n)2^{n+2^n}) \subseteq DTIME(2^{2^n})$.

However, $lim_{n\to\infty} \frac{2^{2^n} log(2^{2^n})}{(n+2^n)2^{n+2^n}} = 0$, contradicting the

time-hierarchy theorem. Here note that 2^{2^n} and

$(n+2^n)2^{n+2^n}$ are both fully time-constructible functions.