Preliminaries

We will start T04 by asking some quick questions from Lecture 5 (and a bit of Lecture 6).

- Write Max-Flow as a Linear Program. Again, are you going to solve Max-Flow that way?
- What is the Max-Flow of the (random, but reasonably small) Flow Network drawn in class (using [https://visualgo.net/en/maxflow](https://visualgo.net/en/maxflow))? Run (standard) Ford Fulkerson’s algorithm manually first before executing the animation to show your tutor/lecturer that you have fully understand the algorithm.

Discussion Points

**Q1:** Show how to use (standard) Ford-Fulkerson’s algorithm to find a maximum sized matching on Bipartite Graph, or formally known as the Max-Cardinality-Bipartite-Matching (MCBM). Prove that the result is a matching, and that it is the maximum-sized matching. Analyze the running time of your algorithm. Note that we have seen the topic of Graph Matching briefly in Lecture 1 (Deterministic Vertex Cover-2), Lecture 4 (part of Christofides’s algorithm), and will properly come back to this topic on Lecture 8.

**Q2a:** Assume that you have an undirected graph $G = (V, E)$ with a source vertex $s$ and a target vertex $t$ (the graph is unweighted). Give an algorithm that finds the maximum number of edge-disjoint paths from $s$ to $t$. Two paths $P_1$ and $P_2$ are called edge-disjoint if they do not share any edges—but they may share a vertex. (Hint: Use (standard) Ford-Fulkerson’s algorithm.) What is the running time of your algorithm?

**Q2b:** What if you want to find the maximum number of vertex-disjoint paths from $s$ to $t$ instead? Two paths are called vertex-disjoint if they do not share any vertex.

**Q3:** Please read [https://uva.onlinejudge.org/external/113/11380.pdf](https://uva.onlinejudge.org/external/113/11380.pdf) and give the correct Flow Graph to solve this problem.

Post Tutorial

Students can discuss PS2 solutions with the lecturer/tutor if you still have doubts (PS2 version V maybe requires too much time to explain if you have not taken CS3233 or involved in ICPC/IOI before). Rais will showcase his recent work on PS2: [https://visualgo.net/en/steinertree](https://visualgo.net/en/steinertree) to the class.