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CS3233
Competitive Programming

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Week 05 – Problem Solving Paradigms
(Dynamic Programming 2)
Outline

• Mini Contest #4 + Break + Discussion + Admins
• A simple DP problem to refresh our memory (Section 3.5.3)
• DP and its relationship with (implicit) DAG (Section 4.7.1)
  – These are CS2020/CS2010 materials
    • Those who have not taken either module must consult Steven separately
• DP on Math Problems (Section 5.4 and 5.6)
• DP on String Problems (Section 6.5)
• More DP techniques (Section 8.3)
• Pointers to other DP techniques in CP2.9
More DP Problems in Chapter 3-4-5-6-8-9 of CP2.9 Book

NON CLASSICAL DP PROBLEMS
Non Classical DP Problems

- **My** definition:
  - Not the pure form (or simple variant) of 1D/2D Max Sum, LIS, 0-1 Knapsack/Subset Sum, Coin Change, TSP where the DP **states** and **transitions** can be “memorized”
  - Requires **original** formulation of DP states and transitions
  - Throughout this lecture, we will talk mostly in **DP terms**
    - **State** (to be precise: “distinct state”)
    - **Space Complexity** (i.e. the number of distinct states)
    - **Transition** (which entail overlapping sub problems)
    - **Time Complexity** (i.e. num of distinct states * time to fill one state)
Refresher: Cutting Sticks

- State: index \((l, r)\) where \(l, r \in [0..n+1]\) and \(l < r\)
  - Q: Why these two parameters?
- Space Complexity: \(O(n^2)\) distinct states
- Transition: Try all possible cutting points \(i\) between \(l\) and \(r\),
  - i.e. cut \((l, r)\) into \((l, i)\) and \((i, r)\) with cost \((A[r] - A[l])\)
- Time Complexity: There are \(O(n)\) possible cutting points, thus overall \(O(n^2 \times n) = O(n^3)\)

\[ A = \{25, 50, 75\} \]
\[ n = 3, \ L = 100 \]
DP on DAG

Overview

• Dynamic Programming (DP) has a close relationship with (usually implicit) Directed Acyclic Graph (DAG)
  – The **states** are the **vertices** of the DAG
  – Space complexity: Number of vertices of the DAG
  – The **transitions** are the **edges** of the DAG
    • Logical, since a recurrence is always **acyclic**
  – Time complexity: Number of edges of the DAG
  – Top-down DP: Process each vertex just once via **memoization**
  – Bottom-up DP: Process the vertices in **topological order**
    • Sometimes, the topological order can be written by just using simple (nested) loops
The Injured Queen Problem

• Like N-queens problem, but the queens are “injured” (can only attack the current column but acts as king otherwise)
• With some of $K$ ($0 \leq K \leq N$) injured queens positions have been predetermined, count how many possible arrangements of the other $(N-K)$ queens so that no two queens attack each other?
DP on Math Problems

• Some well-known mathematic problems involves DP
  – Some combinatorics problem have recursive formulas which entail overlapping subproblems
    • e.g. those involving Fibonacci number, \( f(n) = f(n - 1) + f(n - 2) \)
  – Some probability problems require us to search the entire search space to get the required answer
    • If some of the sub problems are overlapping, use DP, otherwise, use complete search
  – Mathematics problems involving **static** range sum/min/max!
    • Use dynamic tree DS for dynamic queries
Dice Throwing

• $n$ common cubic dice are thrown ($1 \leq n \leq 24$)
• What is the probability that the sum of all thrown dices is at least $x$? ($0 \leq x \leq 150$)
• Basic probability = $\#$ events / $|\text{sample space}|$
• To compute the $|\text{sample space}|$ is easy: It is $6^n$
• The $\#$ events is harder to compute...
DP on String Problems

- Some string problems involves DP
  - Usually, we do not work with the string itself
  - But we work with the integer indices to represent suffix/prefix/substring
    - Reason: Too costly to pass (sub)strings around as function parameters
String Partition

• There are many ways to split a string of digits into a list of non-zero-leading (0 itself is allowed) 32-bit *signed* integers
  – What is the maximum sum of the resultant integers if the string is split appropriately? Examples:
    • 1234554321
      – 1234554321 < 2147483647, so the answer is 1234554321 itself
    • 5432112345
      – 5432112345 > 2147483647, thus 5432112345 must be partitioned
      – There are two ways to partition 5432112345
        » 5 + 432112345 = 432112350, or
        » 543211234 + 5 = 543211239 ⇐ the answer
    • 121212121212
      – 121212121212 > 2147483647, thus 121212121212 must be partitioned
      – The answer is: 1 + 2121212121 + 2 = 2121212124
DP with bitmask

• Bitmask technique can be used to represent *lightweight set of Boolean* (up to $2^{64}$ if using unsigned long long)
• This is important if one of the DP parameter is a “small set”
• We have seen this form earlier in DP-TSP
• One other useful application (there are many others):
  – *Finding min weighted perfect matching in small general graph*
Forming Quiz Teams

Can you spot one more possible grouping? (which is not optimal)

<table>
<thead>
<tr>
<th>N = 2</th>
<th>Not Optimal</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost = 8.60 + 7.61 = 16.21</td>
<td>Cost = 2.00 + 2.83 = 4.83</td>
</tr>
</tbody>
</table>
Common DP States (1)

- Position:
  - Original problem: \([x_1, x_2, \ldots, x_n]\)
    - Can be sequence (integer/double array), can be string (char array)
  - Sub problems, break the original problem into
    - Sub problem and Prefix: \([x_1, x_2, \ldots, x_{n-1}] + x_n\)
    - Suffix and sub problem: \(x_1 + [x_2, x_3, \ldots, x_n]\)
    - Two sub problems: \([x_1, x_2, \ldots, x_i] + [x_{i+1}, x_{i+2}, \ldots, x_n]\)
  - Example: 1D Max Sum, LIS, etc
Common DP States (2)

• Positions:
  – This is similar to the previous slide
  – Original problem: \([x_1, x_2, \ldots, x_n] \text{ and } [y_1, y_2, \ldots, y_n]\)
    • Can be two sequences/strings
  – Sub problems, break the original problem into
    • Sub problem and prefix: \([x_1, x_2, \ldots, x_{n-1}] + x_n\) and \([y_1, y_2, \ldots, y_{n-1}] + y_n\)
    • Suffix and sub problem: \(x_1 + [x_2, x_3, \ldots, x_n]\) and \(y_1 + [y_2, y_3, \ldots, y_n]\)
    • Two sub problems: \([x_1, x_2, \ldots, x_i] + [x_{i+1}, x_{i+2}, \ldots, x_n]\) and \([y_1, y_2, \ldots, y_i] + [y_{i+1}, y_{i+2}, \ldots, y_n]\)
  – Example: String Alignment/Edit Distance, LCS, Matrix Chain Multiplication (MCM), etc
  – PS: Can also be applied on 2D matrix, like 2D Max Sum, etc
Tips: When to Choose DP

• Default Rule:
  – If the given problem is an **optimization** (max/min) or **counting** problem
    • Problem exhibits optimal sub structures
    • Problem has overlapping sub problems

• In ICPC/IOI:
  – If actual solutions are not needed (only final values asked)
    • If we must compute the solutions too, a more complicated DP which stores *predecessor information* and *some backtracking* are necessary
  – The number of distinct sub problems is small enough (< 1M)
    and you are not sure whether greedy algorithm works (why gamble?)
  – Obvious overlapping sub problems detected :O
Dynamic Programming Issues (1)

• Potential issues with DP problems:
  – They may be disguised as (or looks like) non DP
    • It looks like greedy can work but some cases fails...
      – e.g. problem looks like a shortest path with some constraints on graph, but the constraints fail greedy SSSP algorithm!
  – They may have subproblems but not overlapping
    • DP does not work if overlapping subproblems not exist
      – Anyway, this is still a good news as perhaps Divide and Conquer technique can be applied
Dynamic Programming Issues (2)

– Optimal substructures may not be obvious
  1. Find correct “states” that describe problem
     – Perhaps extra parameters must be introduced?
  2. Reduce a problem to (smaller) sub problems (with the same states) until we reach base cases

– There can be more than one possible formulation
  • Pick the one that works!
DP Problems in ICPC (1)

• The number of problems in ICPC that must be solved using DP are growing!
  – At least one, likely two, maybe three per contest...

• These new problems are not the classical DP!
  – They require deep thinking...
  – Or those that look solvable using other (simpler) algorithms but actually must be solved using DP
  – Do not think that you have “mastered” DP by only memorizing the classical DP solutions!
DP Problems in ICPC (2)

• In 1990ies, mastering DP can make you “king” of programming contests...
  – Today, it is a must-have knowledge...
  – So, get familiar with DP techniques!

• By mastering DP, your ICPC rank is probably:
  – from top ~[25-30] (solving 1-2 problems out of 10)
    • Only easy problems
  – to top ~[15-20] (solving 3-4 problems out of 10)
    • Easy problems + brute force + DP problems
For Week 07 homework 😊
(You can do this over recess week too)

BE A PROBLEM SETTER
Be a Problem Setter

• Problem Solver:
  A. Read the problem
  B. Think of a good algorithm
  C. Write ‘solution’
  D. Create tricky I/O
  E. If WA, go to A/B/C/D
  F. If TLE/MLE, go to A/B/C/D
  G. If AC, stop 😊

• Problem Setter:
  A. Write a good problem
  B. Write good solutions
     • The correct/best one
     • The incorrect/slower ones
  C. Set a good secret I/O
  D. Set problem settings

• A problem setter must think from a different angle!
  – By setting good problems, you will simultaneously be a better problem solver!!
Problem Setter Tasks (1)

- **Write a good problem**
  - Options:
    - Pick an algorithm, then find problem/story, or
    - Find a problem/story, then identify a good algorithm for it (harder)
  - Problem description must not be ambiguous
    - Specify input constraints
    - Good English!
    - Easy one: longer, Hard one: shorter!

- **Write good solutions**
  - Must be able to solve your own problem!
    - To set hard problem, one must increase his own programming skill!
  - Use the best possible algorithm with lowest time complexity
    - Use the inferior ones ‘that barely works’ to set the WA/TLE/MLE settings...
Problem Setter Tasks (2)

- Set a good secret I/O
  - Tricky test cases to check AC vs WA
  - Usually ‘boundary case’
  - Large test cases to check AC vs TLE/MLE
    - Perhaps use input generator to generate large test case, then pass this large input to our correct solution

- Set problem settings
  - Time Limit:
    - Usually 2 or 3 times the timings of your own best solutions
      - Java is slower than C++!
  - Memory Limit:
    - Check OJ setting
  - Problem Name:
    - Avoid revealing the algorithm in the problem name
FYI: Be A Contest Organizer

• Contest Organizer Tasks:
  – Set problems of *various* topic
    • Better set by >1 problem setter
  – Must balance the difficulty of the problem set
    • Try to make it fun
    • Each team solves some problems
    • Each problem is solved by some teams
    • No team solve all problems
      – Every teams must work until the end of contest
More References

• **Competitive Programming 2.9**
  – Section 3.5, 4.7.1, 5.4, 5.6, 6.5, 8.3, and parts of Ch9
• **Introduction to Algorithms**, p323-369, Ch 15
• **Algorithm Design**, p251-336, Ch 6
• **Programming Challenges**, p245-267, Ch 11
• Best is practice & more practice!

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