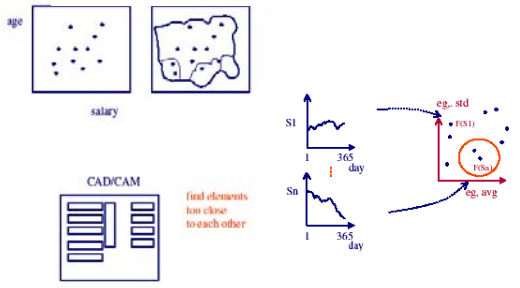
Multidimensional (Spatial) Indexing

Motivation

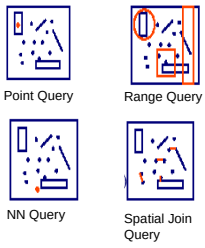
- Many applications of databases manipulate geographical (2-d) data. Others involve large number of dimensions
- Examples:
  - location of restaurants in a city.
  - Map data: zones, county lines, rivers, lakes, etc. (Data has spatial extent)
  - Sales information described by store, day, item, color, size, etc. Sale = point in multidimensional space.
  - Student described by age, zipcode, marital status.

Applications with Multi-Dimensional Data



Types of Queries

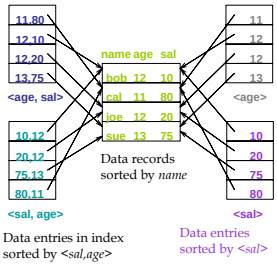
- Point queries
- Range Query: "find all McDonald restaurants within a given region".
- Nearest Neighbor Query: Find the nearest McDonald to my house
- Partial match queries
- Spatial join ("all pairs" queries)



Multi-attribute Indexes

- *Composite Search Keys*: Search on a combination of fields.
  - Equality query: Every field value is equal to a constant value. E.g. wrt <sal,age> index:
    - age=12 & sal =75
  - Range query: Some field value is not a constant. E.g.:
    - age=12 & sal > 10 (use <age, sal>)
    - age < 12 & sal = 10 (use <age,sal> may fetch more records than desired)
- Data entries in index sorted by search key to support range queries.
  - Lexicographic order, or
  - Spatial order.

Examples of composite key indexes using lexicographic order.



Bitmap Indexes

- Bitmap indices are a special type of index designed for efficient querying on multiple keys
- Records in a relation are assumed to be numbered sequentially
- Given a number *n* it must be easy to retrieve record *n* (Particularly easy if records are of fixed size)
- Applicable on attributes that take on a relatively small number of distinct values
  - E.g. gender, country, state, …
  - E.g. income-level (income broken up into a small number of levels such as 0-9999, 10000-19999, 20000-50000, 50000- infinity)
- A bitmap is simply an array of bits

Use of Bitmap Indexes: Example

- In its simplest form, a bitmap index on an attribute has a bitmap for *each value* of the attribute
  - Bitmap has as many bits as records
  - In a bitmap for value v, the bit for a record is 1 if the record has the value v for the attribute, and is 0 otherwise
  - Size = nm bits where n is the #records, m is the #distinct values

record number	*name*	*gender*	*address*	*income-level*
0	John	m	Perryridge	L1
1	Diana	f	Brooklyn	L2
2	Mary	f	Jonestown	L1
3	Peter	m	Brooklyn	L4
4	Kathy	f	Perryridge	L3

Bitmaps for *gender*

m	10010
f	01101

Bitmaps for *income-level*

L1	10100
L2	01000
L3	00001
L4	00010
L5	00000

Bitmap Indexes (Cont.)

- Queries are answered using logical (bitwise) operations
  - Intersection (and)
  - Union (or)
  - Complementation (not)
- Each operation takes two bitmaps of the same size and applies the operation on corresponding bits to get the result bitmap
  - Males with income level L1
    - 10010 AND 10100 = 10000
    - Can then retrieve required tuples
    - Counting number of matching tuples is even faster
- Range queries?
  - Age IN [30,40] AND Salary IN [10k,20k]

Compressed Bitmaps

- If n and m are large, then nm bits may incur high I/O
- Compress the bitmap – run-length encoding
  - A sequence of *i* 0's followed by a 1 (run) is represented by some binary encoding of the integer *i*
  - A number *i* is represented by ($\log_2 i$-1) 1-bit (indicates the number of bits required to represent *i*) and a single 0, followed by its binary value
    - E.g., 13 = 1101 (binary) is represented as

      111 0 1101

      log13-1 13
    - Exceptions: *i* = 0 is 00; *i* = 1 is 01
  - Every run incurs 2 $\log_2 i$ bits

Compressed Bitmap (Cont.)

- Consider 000000000000011 0001
  - The encoded sequence is …
- Now consider 000000010000 (i.e., n = 12)
- What is the compressed bitmap?
- Decode 110111
  - What about the (missing) 0's?

Operating on Compressed Bitmap

- Need to decode first, then perform the bitwise operations
- But can be done incrementally
- Suppose we ORed encodings:

0 0 1 1 0 1 1 1 1 1 0 1 1 1

0 7

0 00000001

OUTPUT: 0

Operating on Compressed Bitmap

- Need to decode first, then perform the bitwise operations
- But can be done incrementally
- Suppose we ORed encodings:

0 0 1 1 0 1 1 1 1 1 0 1 1 1

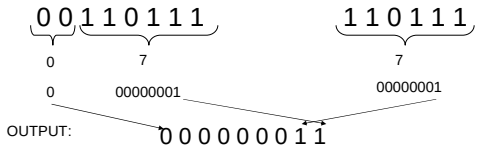
0 7 7

0 00000001 00000001

OUTPUT: 0 0 0 0 0 0 0 1

Operating on Compressed Bitmap

- Need to decode first, then perform the bitwise operations
- But can be done incrementally
- Suppose we ORed encodings:



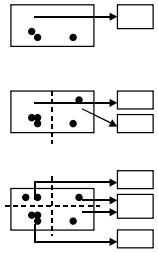
Why spatial index methods (SAMs)?

- B-tree & hash tables
  - Guarantee the number of I/O operations is respectively logarithmic and constant with respect to the collection's size
  - Index a collection on a key
  - Rely on a total order on the key domain, the order of natural numbers, or the lexicographic order on strings
- There is no such total order for multidimensional objects and geometric objects with spatial extent
- SAMs were designed to try as much as possible to preserve spatial object proximity

Multidimensional Indexing Structures

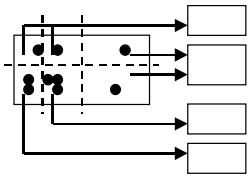
- Space-Based structures:
  - Partition the embedding Space into rectangular cells
  - Independent from the distribution of the objects
  - Objects are mapped to the cells based on some geometric criterion
  - Eg: Grid file, Buddy-tree, KDB-tree
- Data-Based structures:
  - Organize by partitioning the set of objects based on spatial proximity such that each group can fit into a page
  - Adapt to the objects' distribution
  - Eg. R-tree, R* tree, R+ tree
- Mapping
  - Transform the data into lower dimensional space
  - E.g., space filling curve

Grid File: A Space-based Approach



- Start with one bucket for the whole space.
- Select dividers along each dimension. Partition space into cells
- Dividers cut all the way

Grid File

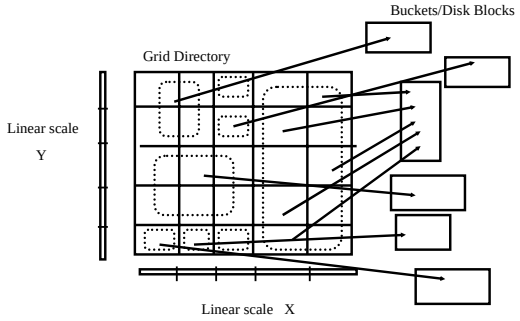


- Each cell corresponds to 1 disk page.
- Many cells can point to the same page.
- Cell directory potentially exponential in the number of dimensions

Grid File Implementation

- Dynamic structure using a grid directory
  - Grid array: a 2 dimensional array with pointers to buckets (this array can be large, disk resident) G(0,…, nx-1, 0, …, ny-1)
  - Linear scales: Two 1 dimensional arrays that used to access the grid array (main memory) X(0, …, nx-1), Y(0, …, ny-1)

Example



Grid File Search

- Exact Match Search: at most 2 I/Os assuming linear scales fit in memory.
  - First use linear scales to determine the index into the cell directory
  - access the cell directory to retrieve the bucket address (may cause 1 I/O if cell directory does not fit in memory)
  - access the appropriate bucket (1 I/O)
- Range Queries:
  - use linear scales to determine the index into the cell directory.
  - Access the cell directory to retrieve the bucket addresses of buckets to visit.
  - Access the buckets.
- Nearest Neighbor Queries:
  - Search the bucket corresponding to the cell in which the query point P belongs
  - If we find at least one point, we have a candidate Q for the NN
  - However, it is possible that there are points in adjacent buckets that are closer to P than Q is; in this case, we need to search these buckets.

Grid File Insertions

- Determine the bucket into which insertion must occur.
- If space in bucket, insert.
- Else, split bucket (alternatively: use overflow buckets?)
  - how to choose a good dimension to split?
  - ans: create convex regions for buckets.
- If bucket split causes a cell directory to split do so and adjust linear scales.
  - When does the cell directory not split?
- insertion of these new entries potentially requires a complete reorganization of the cell directory---expensive!!!

Grid File Deletions

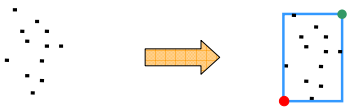
- Deletions may decrease the space utilization. Merge buckets
- We need to decide which cells to merge and a merging threshold
- Buddy system and neighbor system
  - A bucket can merge with only one *buddy* in each dimension
  - Merge adjacent regions if the result is a rectangle

R-Trees: A Data-based Approach

- **R-trees**
  - N-dimensional extension of B^+-trees
    - Height-balanced, all leaf nodes appear at the same level, each node has between t and M entries
  - useful for indexing sets of points, rectangles and other polygons.
- Basic idea: generalize the notion of a one-dimensional interval associated with each B+ -tree node to an N-dimensional interval, that is, an N-dimensional rectangle.
- Will consider only the two-dimensional case ($N = 2$)
  - generalization for $N > 2$ is straightforward

Bounding Rectangle

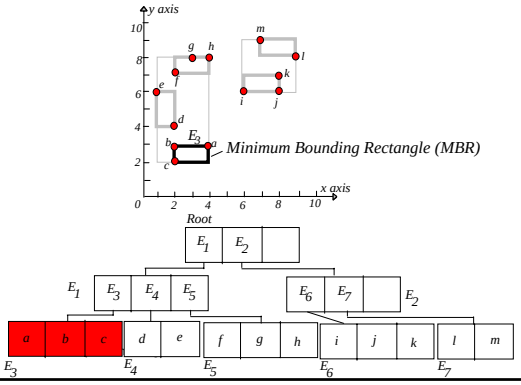
- Suppose we have a cluster of points in 2-D space...
  - We can build a "box" around points. The smallest box (which is axis parallel) that contains all the points is called a Minimum Bounding Rectangle (MBR)
    - also known as minimum bounding box



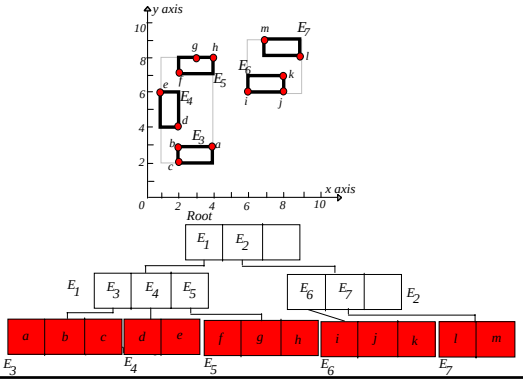
MBR = {(L.x,L.y)(U.x,U.y)}

Note that we only need two points to describe an MBR, we typically use lower left, and upper right.

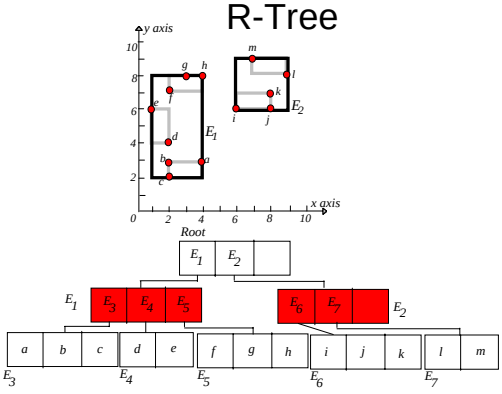
R-Tree: Clustering by Proximity



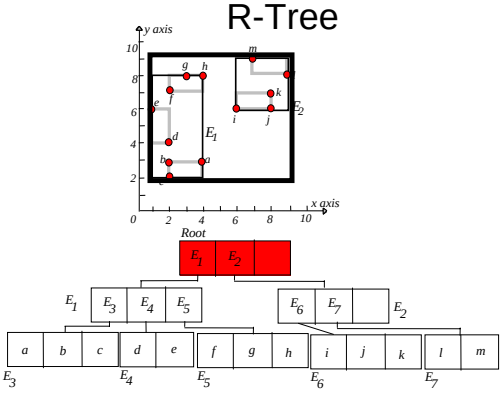
R-Tree



R-Tree



R-Tree

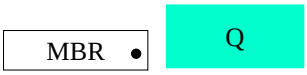


R Trees (Cont.)

- A rectangular **bounding box (a.k.a. MBR)** is associated with each tree node.
  - Bounding box of a leaf node is a minimum sized rectangle that contains all the points associated with the leaf node.
  - The bounding box associated with a non-leaf node contains the bounding box associated with all its children.
  - Bounding box of a node serves as its key in its parent node (if any)
  - *Bounding boxes of children of a node are allowed to overlap*

R-Tree structure

- leaf entry = <object>
  - An object may have extent.
- index entry = <MBR, ptr to child node>
  - MBR of all objects in the subtree.



- Observation: if range query, Q, does not intersect an MBR, no object in the sub-tree is inside Q.

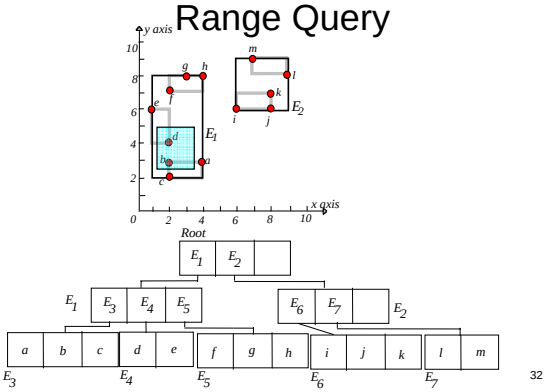
Range query

Range Query: Given a query box Q, find all points enclosed by Q

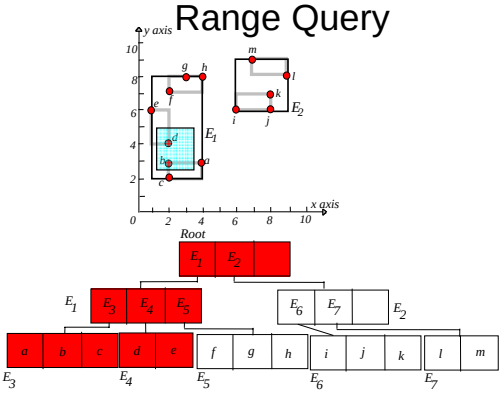
Start at root.
1. If current node is non-leaf, for each entry <E, ptr>, if ***box*** E overlaps Q, search subtree identified by ptr.
2. If current node is leaf, for every object in the leaf page, report if contained in Q.

Can be very inefficient in worst case since multiple paths may need to be searched but works acceptably in practice.

Range Query



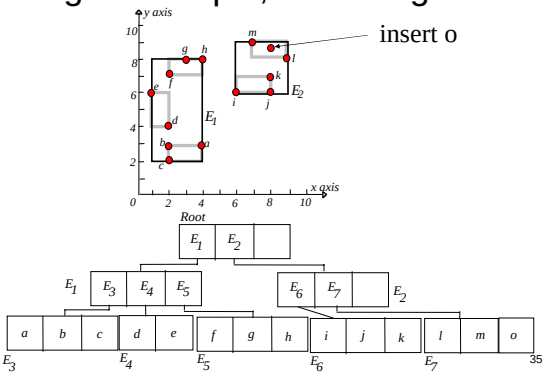
Range Query



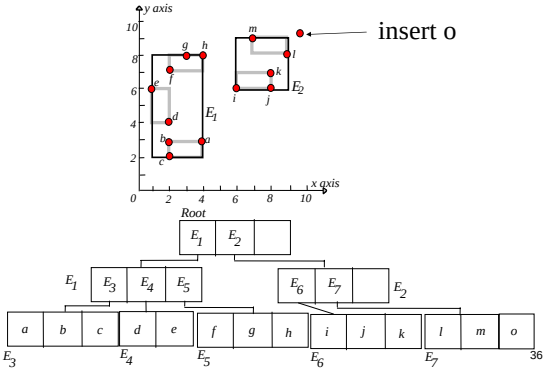
Insert object o

- Start at root and go down to "best-fit" leaf L.
  - Go to child whose box needs least enlargement to cover o; resolve ties by going to smallest area child.
- If best-fit leaf L has space, insert entry, adjust bounding boxes starting from the leaf upwards
- Otherwise, split L into L1 and L2.
  - Adjust entry for L in its parent so that the box now covers (only) L1.
  - Add an entry (in the parent node of L) for L2. (This could cause the parent node to recursively split.)

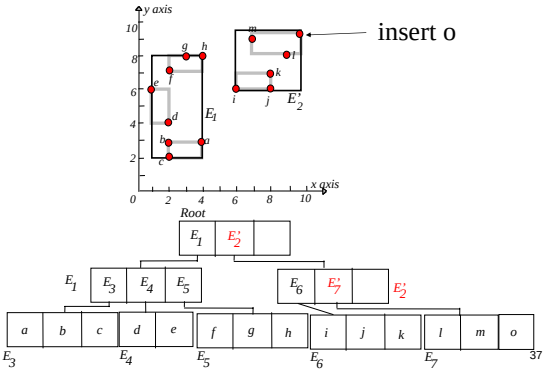
E.g. 1: no split, no enlargement



E.g. 2: no split, but enlargement



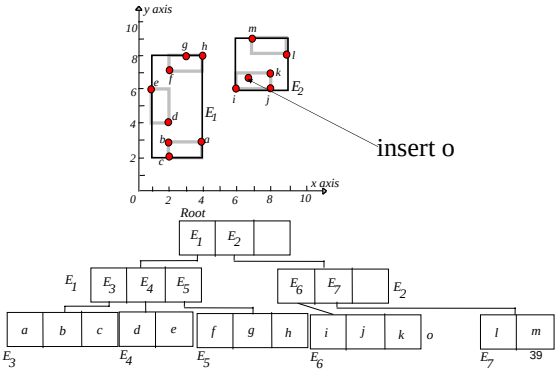
E.g. 2: no split, but enlargement



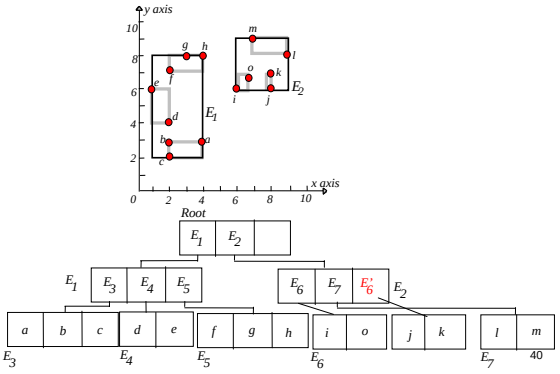
Insert object o

- Start at root and go down to "best-fit" leaf L.
  - Go to child whose box needs least enlargement to cover o; resolve ties by going to smallest area child.
- If best-fit leaf L has space, insert entry, adjust bounding boxes starting from the leaf upwards
- Otherwise, split L into L1 and L2.
  - Adjust entry for L in its parent so that the box now covers (only) L1.
  - Add an entry (in the parent node of L) for L2. (This could cause the parent node to recursively split.)

E.g. 3: split



E.g. 3: split



Splitting a Node During Insertion

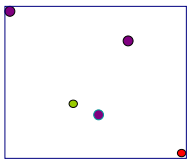
- The entries in node L plus the newly inserted entry must be distributed between L1 and L2.
- Goal is to reduce likelihood of both L1 and L2 being searched on subsequent queries.
- Idea: Redistribute so as to minimize area of L1 plus area of L2.

Exhaustive algorithm is too slow; quadratic and linear heuristics are described in the paper.

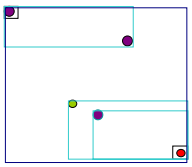
Quadratic Split

- **Quadratic split** divides the entries in a node into two new nodes as follows
  1. Find pair of entries with "maximum separation"
     - that is, the pair such that the bounding box of the two would have the maximum wasted space (area of bounding box – sum of areas of two entries)
  2. Place these entries in two new nodes
  3. Repeatedly find an entry and assign it to one of the nodes
     - Calculate d_1 = the area increase required in the MBR of one node to include the entry. Calculate d_2 analogously for the other node
     - Choose the entry with the maximum difference between d_1 and d_2
     - Add It to the node whose MBR will have to be enlarged least to accommodate It. Resolve ties by adding the entry to the node with the smallest area, then to the one with the fewer entries, then to either
  4. Stop when half the entries have been added to one node
     - Then assign remaining entries to the other node

Node Splitting R-trees



Node Splitting R-trees



Deleting in R-Trees

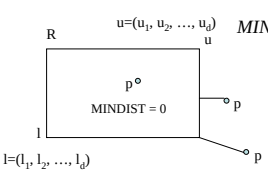
- Deletion of an entry in an R-tree done much like a B^+-tree deletion.
  - In case of underfull node, borrow entries from a sibling if possible, else merging sibling nodes
  - Alternative approach removes all entries from the underfull node, deletes the node, then reinserts all entries

NN search: Best-First Algorithm

- Global order
  - Use only MINDIST
  - Maintain distance to all entries in a common Priority Queue
  - Repeat
    - Inspect the next MBR in the list
    - Add the children to the list and reorder
  - Until all remaining MBRs can be pruned

MINDIST

- MINDIST(P, R) is the minimum distance between a point P and a rectangle R
- If the point is inside R, then MINDIST=0
- If P is outside of R, MINDIST is the distance of P to the closest point of R (one point of the perimeter)

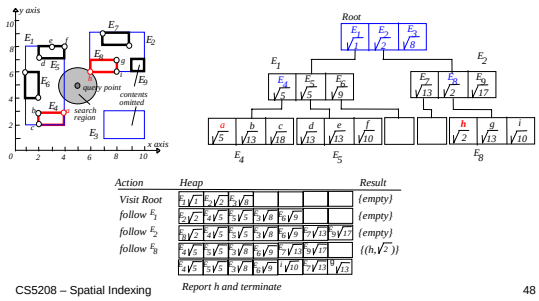


$$MINDIST(P,R) = \sqrt{\sum_{i=1}^{d} (p_i - r_i)^2}$$

$r_i = l_i$ if $p_i < l_i$
$= u_i$ if $p_i > u_i$
$= p_i$ otherwise

$$\forall o \in R, MINDIST(P,R) \le \|(P,o)\|$$

Nearest Neighbor Search (NN) with R-Trees



The NN Search Algorithm

Initialize PQ (priority queue)
InesrtQueue(PQ, Root)
While not IsEmpty(PQ)
  R= Dequeue(PQ)
  If R is an object
    Report R and exit (done!)
  If R is a leaf page node
    For each O in R, compute the Actual-Dists, InsertQueue(PQ, O)
  If R is an index node
    For each MBR C, compute MINDIST, insert into PQ

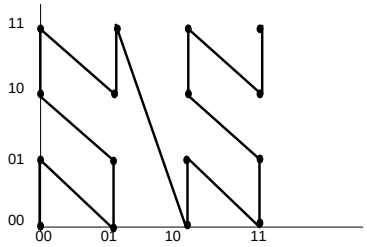
K-NN search

- Keep the sorted buffer of at most k current nearest neighbors
- Pruning is done using the k-th distance

Space Filling Curves: Mapping Based Methods

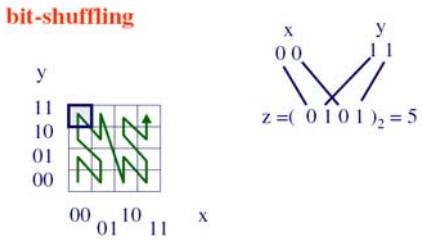
- Assumption: att. values can be represented with some fixed # of bits
- Space domain on each dimension: 2^k values
- Linearize the domain
- Each point can be represented by a single dimensional value

Z-ordering



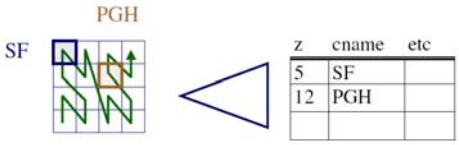
Z-ordering

- The z-value is obtained by interleaving the bits



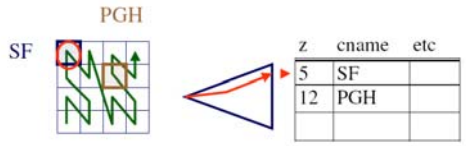
Z-ordering

- z-values can be indexed using B+-trees
  - All commercial systems support B+-tree (no coding/debugging, concurrency and recovery support, etc)



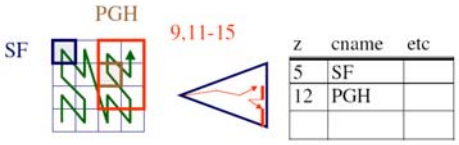
Z-ordering

- Point queries: e.g., *find city at (0,3)*
- Find z-value; search B-tree



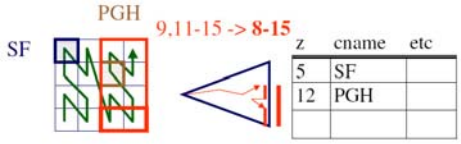
Z-ordering

- Range queries?
- Compute ranges of z-values; use B-tree



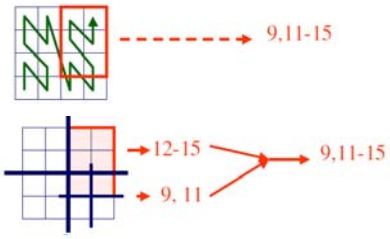
Z-ordering

- Range queries?
- Compute ranges of z-values; use B-tree
- To reduce # of qualifying of ranges: Augment the query
- What's the overhead?



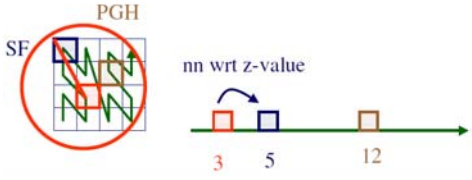
Z-ordering

- Range queries - how to break a query into ranges?
- Recursively, quadtree-style; decompose only non-full quadrants



Z-ordering

- k-nn queries, e.g., 1-nn:
- Traverse B-tree; find nn wrt z-values and … ask a range query.



Summary

- Many applications operate on multi-dimensional data
- Advanced techniques are required
- Paradigm
  - Space-partitioning
  - Data-partitioning
  - Mapping