Relational Algebra Equivalences

- Allow us to choose different join orders and to ‘push’ selections and projections ahead of joins.
- Rules on joins, cross products and union

\[ R \bowtie S = S \bowtie R \]
\[ (R \bowtie S) \bowtie T = R \bowtie (S \bowtie T) \]
\[ R \times S = S \times R \]
\[ (R \times S) \times T = R \times (S \times T) \]
\[ R \cup S = S \cup R \]
\[ R \cup (S \cup T) = (R \cup S) \cup T \]

Rules: Selects

\[ \sigma_{p_1 \land p_2}(R) = \sigma_{p_1}( \sigma_{p_2}(R)) \]
\[ \sigma_{p_1 \lor p_2}(R) = [ \sigma_{p_1}(R) \cup [ \sigma_{p_2}(R) ] \]

Rules: Project

Let: \( X = \) set of attributes
\( Y = \) set of attributes
\( XY = X \cup Y \)
\[ \pi_{xy}(R) = \pi_x[ \pi_y(R) ] \]

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\[ \pi_{xy}(R) = \pi_x[ \pi_y(R) ] \]
\[ \pi_x(R) = \pi_x[ \pi_y(R) ] \text{ if } y \text{ contains } x \]

Rules: \( \sigma + \bowtie \) combined

Let \( P = \) predicate with only \( R \) attributes
\( Q = \) predicate with only \( S \) attributes
\( M = \) predicate with only \( R,S \) attributes
\[ \sigma_p(R \bowtie S) = [ \sigma_p(R) ] \bowtie S \]
\[ \sigma_q(R \bowtie S) = R \bowtie [ \sigma_q(R) ] \]

More Rules

Let \( x = \) subset of \( R \) attributes
\( z = \) attributes in predicate \( P \)
\( \pi_x[ \sigma_p(R) ] = \pi_x \{ \sigma_p[ \pi_{xz}(R) ] \} \)
More Rules

Let \( x = \) subset of \( R \) attributes
\( y = \) subset of \( S \) attributes
\( z = \) intersection of \( R, S \) attributes

\[
\pi_{xy}(R \bowtie S) = \\
\pi_{xy}\left(\pi_{xz}(R) \bowtie \pi_{yz}(S)\right)
\]

More Rules

\[
\pi_{xy}\left\{\sigma_p (R \bowtie S)\right\} = \\
\pi_{xy}\left\{\sigma_p \left[\pi_{xz'}(R) \bowtie \pi_{yz'}(S)\right]\right\}
\]
\( z' = z \cup \{\text{attributes used in } P\} \)