Take-home exercises:
(You do NOT need to submit these questions!)

The **Fibonacci** sequence F is defined as follows:

$$F(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F(n-1) + F(n-2) & \text{for } n > 1 \end{cases}$$

So, the first few Fibonacci numbers are: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, …

T1. It is known that $\boldsymbol{F_{k-1} \times F_{k+1} - F_k^2 = (-1)^k}$ for $k \geq 1$. For example, $F_5 \times F_7 - F_6^2 = 5 \times 13 - 8^2 = 1$; $F_6 \times F_8 - F_7^2 = 8 \times 21 - 13^2 = -1$. Write a program to verify this for values of k in the range [1,20].

T2. It is known that $\boldsymbol{F_{mk}}$ is a multiple of $\boldsymbol{F_k}$ for $k, m \geq 1$. For example, $F_{15} = 610$ is a multiple of $F_5 = 5$ and $F_3 = 2$. Write a program to verify this for values of m and k in the range [1,4].

T3. It is known that $\mathbf{GCD(F_{\mathit{j}},F_{\mathit{k}}) = F_{GCD(\mathit{j},\mathit{k})}}$. For example, $\mathrm{GCD}(\mathrm{F}_6,\mathrm{F}_{12}) = \mathrm{GCD}(8,144) = 8$ and $\mathrm{F}_{\mathrm{GCD}(6,12)} = \mathrm{F}_6 = 8$. Write a program to verify this for values of j and k in the range [1,20].

T4. Prove the following claim

If z = GCD(x, y), then there is no $w > z$ that divides both x and y.

T5. Prove the following claim

If w divides x and y., then it divides $x - y$.

T6. Verify that the following variant of GCD algorithm, BinaryGCD, is correct:

(a) if $x = y$ then return x.

(b) if x and y are even then return 2 × BinaryGCD(x/2, y/2).

(c) if x is even and y is odd then return BinaryGCD(x/2, y).

(d) if x is odd and y is even then return BinaryGCD(x, y/2).

(e) if x and y are odd then return BinaryGCD($|x - y|/2$, min{x, y}).