An Analytical Model for the Dimensioning of a GPRS/EDGE Network with a Capacity Constraint on a Group of Cells

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ABSTRACT

This paper is a contribution to the generic problem of having simple and accurate models to dimension radio cells with data traffic on a GPRS or EDGE network. It addresses the issue of capacity limitation in a given cell due to coupling with other cells because of a central equipment or transmission link of limited capacity. A mobile station can't access the requested resource although it is alone in a cell. The traffic on other coupled cells leads to reach the global capacity limit. Our objective is to avoid the derivation of any multi-dimensional Markovian (or semi-Markovian) model, where each dimension corresponds to a given cell of the system. Such direct extensions would be of non-manageable complexity. Instead we derive an analytical model that captures in an aggregate way the coupling between cells. We show that the performance parameters of the GPRS/EDGE network can be derived quickly and with a very good accuracy. Finally, as our modeling framework allows very fast computations, we show how to use it to perform complex iterative dimensioning studies.

Categories and Subject Descriptors

C.4.4 [**Performance of systems**]: Modeling techniques; C.4.5 [**Performance of systems**]: Performance attributes

General Terms

Design, Performance, Reliability

Keywords

GPRS, EDGE, modeling, Erlang, performance evaluation, dimensionning, Markov chain, global capacity limit

1. INTRODUCTION

GPRS (General Packet Radio Service) is an overlay on GSM networks that allows end-to-end IP-based packet traffic from the terminal to e.g. the Internet. EDGE (Enhanced Data rates for Global Evolution) is an improvement over GPRS whereby the radio modulation scheme is modified to allow higher throughputs thanks to advanced power amplifier and signal processing technologies. Although UMTS is being deployed worldwide, GPRS or EDGE will remain as a key complement for nationwide wireless data coverage. In a GPRS (or EDGE) cell, traffic is split between voice (on circuit) and data (on packet). Data use a few dedicated circuits which are decomposed into 20 ms "blocks" carrying elementary packet traffic. The packet-based traffic is managed by the PCU (Packet Control Unit), a standardized network element in charge of the MAC (Medium Access Control) layer (multiplexing of mobile stations) and RLC (Radio Link Control) layer (decomposition into elementary blocks and retransmission when radio errors occur). The PCU is connected to the SGSN (Serving GPRS Support Node) which manages the end-user mobility and hides it to the external world. It is linked to the edge router, called GGSN (Gateway GPRS Support Node), by an IP tunnel in which traffic is encapsulated. The GGSN is the fixed anchor point to the Internet or service platforms, whereas a user may change SGSN while going from cell to cell. In this end-to-end chain, it is possible to have traffic limitation from an element in charge of managing several cells, typically a PCU or SGSN module or a transmission link. Traditional modeling tools assume a total decoupling i.e. each part of the end-to-end chain is dimensioned in series: cell, transmission link, PCU, etc. The objective of this paper is to develop an effecient model including the central capacity constraint in the radio dimensioning analysis. It will allow not only to efficiently dimension the GPRS/EDGE network but also to better assess the influence of this constraint.

Many papers have tackled the problem of GPRS/EDGE performance analysis. Several papers are based on simulations (see e.g. [19], [21], [22] and [9]). Most of these studies make use of classical wireline traffic assumptions and don't include wireless specific traffic models. Moreover, the accuracy of simulation results is obtained at the expense of long processing time that makes it difficult to obtain a large number of results which are of great use to provide qualitative understanding and numerous quantitative data.

Performance evaluation using analytical models are proposed in [20], [23], [14], [10], [12], [11], [15], [17]. In [17], Ni and Häggman consider a system where a fixed number of channels are reserved for GPRS traffic while the rest of the TDMA (Time Division Multiple Access) is shared with voice calls. The inter-arrivals of GPRS packets as well as the service times are supposed to be exponentially distributed. A

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similar system is studied in [15] but the inter-arrival times are assumed to be geometrically distributed. A complete sharing policy between voice and GPRS traffic (with voice priority) is investigated. When the air interface is congested, GPRS packets are stored in a queue of infinite length. A Quasi-Death-Birth (QDB) model is developed in order to estimate the average waiting time of GPRS packets.

In these papers, GPRS sessions are assumed to be of infinite duration. Finite-length GPRS sessions are investigated in [23], [14], [10], [12] and [11]. In in [23] Vornefeld proposes a Marked Markovian Arrival Process (MMAP) that aims at modeling each mobile station arrival process of data packets. In [14], [10], [12] and [11], the bursty nature of the aggregated GPRS traffic is modeled by means of a two state Modulated Markov Poisson Process (MMPP). In this model, a state ON relates to an active GPRS session while an OFF period corresponds to a handover disruption or a termination of a GPRS call. The performance of the system is analyzed for different bandwidth sharing policies (trunk reservation in [14] and [12], fixed, partial or complete sharing in [11]).

In [5] we have developed a discrete-time Markov chain model for single-cell GPRS/EDGE network engineering. The model captures the main features of the GPRS/EDGE radio resource allocation and assumes an ON/OFF traffic (with infinite sessions) performed by a finite number of users over the cell. The Markov chain is further simplified by Taylor series expansion and a simple and accurate Erlang-like law is obtained. Extensions to finite-length sessions traffic are developed in [2] and [1]. In [2] we show that a simple modification of the traffic parameters allows us to obtain another Erlang-like law for the performance parameters, with a very good accuracy.

All these studies rely on a number of assumptions, and contribute usefully to the understanding of GPRS/EDGE system. However, even when groups of cells are considered (see e.g. [14], [16]), the interactions between cells only come from mobility and thus only modify the traffic characteristics (by introducing handovers and reselections between cells).

In this paper we study the impact of capacity limitation imposed by a centralized component upon a group of cells. We assume that the limitation takes the following form: a total of no more than M_{max} concurrent transfers are allowed in the group of cells. Typically this limit can be a software imposed limitation or it can be an approximate model of the limited processing power of the equipment shared by all the cells which will reject any transfer request when it is reached. M_{max} is known or measured. Our purpose is first to understand whether such limitations have a significant impact on network performance, then it is to provide an efficient multiple-cell dimensioning tool. The task seems a priori quite complex since a natural generalization to multiple-cell systems consists in developing a multi-dimensional Markov chain (or semi-Markovian model) where each dimension corresponds to a given cell. Typically more than 30 cells are considered in a group. Thus, such an extension is of non-manageable complexity. We work around the problem by developing the so-called "aggregate Markov chain model". It approximates in an aggregate way the coupling between a given cell and the other ones which are constrained by the same global capacity limit. We show that the performance parameters can be obtained almost instantaneously with a very good accuracy.

Section 2 first presents the basic assumptions and the main results of the Erlang-like model developed in [5] for the single-cell case. In Section 3 we develop the analytical model for the performance evaluation of the constrained group of cells. Section 4 validates the model by comparison with simulation results. In Section 5 we investigate the influence of the main input parameters on the system behavior. Section 6 finally addresses the model utilization for dimensioning.

2. SINGLE CELL SYSTEM

2.1 System description

Our study is focused on the analysis of the bottleneck i.e. the radio link, studied in a particular cell. We are focused on the downlink, assumed to be the critical resource because of traffic asymmetry. We assume that the allocator fairly shares bandwidth between all active mobile stations. As a short reminder, GPRS or EDGE is a packet overlay on the circuit-based GSM system. With GSM, on each frequency carrier a 200 kHz bandwidth is shared between 8 users. Each user is given a circuit, also called time-slot because it is a Time-Division mutliplexing scheme (TDMA). With GPRS and EDGE, a mobile station can use several time-slots simultaneously to have a higher throughput. However, timeslots may be shared between mobiles with a granularity of 20 ms (a so-called "radio block"). Every 20 ms the PCU allocates the timeslots to the mobiles having an on-going transfer, with a fair sharing scheme (we don't consider here QoS priorities). The throughput depends also on the radio coding scheme, i.e. the amount of overhead in the transmitted data to add error protection against radio conditions.

We make the following assumptions:

- The radio resource is divided into two separate parts according to the so-called Complete Partitioning policy [11], one dedicated to voice and one dedicated to data. Our work only focuses on the data part and assumes that there is a fixed number T of time-slots in the cell that are dedicated to GPRS. Obviously the classical Erlang formulas apply for voice. In [3] we investigate the Partial Partitioning policy (in a single-cell environment) and develop an extension of our basic model [5] that takes into account the preemption of voice over data, only on a given shared part of the TDMA. We finally assume that the T time-slots are using a single TDMA frame. If there are more than one TDMA frame, resources may not be equally dividable among mobiles. It introduces an additional (slight) complexity which is not addressed here.

- All mobiles have the same reception capability. They are "$(d+u)$", where d is the number of time-slots that can be used simultaneously for the downlink traffic and u is the number of time-slots that can be used simultaneously for the uplink traffic. Note that, as we are only interested in the modeling of the downlink traffic, only the parameter d is relevant for the model. Nowadays, most recent mobiles are $(4+1)$ or $(4+2)$ and can thus use at most 4 time-slots simultaneously in downlink.

Our GPRS system is characterized by the following parameters:

- t_B: the system elementary time interval equal to the radio block duration, i.e. $t_B = 20$ ms;

- x_B: the number of data bytes that are transferred during t_B over one time-slot. x_B/t_B is the throughput offered by the RLC/MAC layer to the above LLC (Logical Link Control) transport layer. The value of x_B depends on the radio coding scheme.

  For GPRS we have:

GPRS coding scheme	CS1	CS2	CS3	CS4
x_B (in bytes)	20	30	36	50

  For EDGE we have:

EDGE coding scheme	MCS1	MCS2	MCS3
x_B (in bytes)	22	28	37

MCS4	MCS5	MCS6	MCS7	MCS8	MCS9
44	56	74	112	136	148

  Note that non ideal radio conditions decrease the available throughput by inducing radio block retransmissions: erroneous radio blocks are very efficiently detected in typical radio conditions and they are selectively retransmitted. Throughput reduction can be modeled by multiplying x_B by a reduction factor. A first approximation for this factor is (1-BLER) where BLER is the Block Error Rate (it assumes independent and potentially infinitely retransmitted blocks in error). More precisely, this factor can be found by simulations of the full GPRS/EDGE radio protocol. Furthermore, simulations giving BLER as a function of the of signal to (noise+interference) ratio can provide the link between throughput and the overall radio environment. Note that since our final model will be computationally very fast ,we can include it in a global network modeling tool where traffic generates interferences which in turn generate BLER and throughput degradation. It will be run iteratively, with fixed point methods for instance, to provide the overall network performance picture.

- tbf_{max}: the maximum number of mobiles that can simultaneously have an active downlink TBF (Temporary Block Flow). On a single TDMA, assuming uplink and downlink flows occur concurrently, system specifications give:

$$tbf_{max} = \min(32, 7T, mT) \tag{1}$$

  because of the GPRS system limitations on the signalling capabilities (no more than 32 TFIs (Temporary Flow Identity) per TDMA, 7 USFs (Uplink State Flag) per uplink time-slot); m is an additional settable parameter which describes a minimum throughput per mobile if an admission control scheme is used (no more than m mobiles per time-slot).

2.2 Traffic model

Traffic is modeled as follows. We assume that there is a fixed number N of GPRS mobiles that are sharing the total bandwidth of the cell. Such a finite population assumption is typically used for network planning when geo-marketing data allows to predict the active GPRS population that will be served by the cell. For a network in service, traffic statistics can also provide estimates of this population. At this stage we don't consider the mobility effect where sessions are interrupted during cell transitions. In a sense our approach considers that among the users which change cell while in traffic, there is a balance between incoming and outgoing users and overall session characteristics are not significantly affected. Each of them is doing an ON/OFF traffic with an infinite number of pages:

- ON periods correspond to the download of an element (a WAP (Wireless Application Protocol), a web page or component of it, an email, a file, etc.). Its size is characterized by a discrete random variable X_{on}, with an average value of x_{on} bytes;

- OFF periods correspond to the reading time, which is modeled as a continuous random variable T_{off}, with an average value of t_{off} seconds.

Several results on insensitivity (see e.g. [6], [13] and [7]) have shown for similar systems that the average performance parameters of the model is sensitive to the average traffic parameters rather than their full distribution. Even though we were not able demonstrate that this result holds for the model developed in [5] all the experiments we made [4] tend to prove that the insensibility is still true (or at least a very good approximation). Thus memoryless (exponential or geometric) distributions are the most convenient choices to model traffic. Then we can model complex WAP or web page downloads even though a page is made of several components. Note also that the best practice for end-to-end performance optimization consists in sending pages in a single download, for instance through multipart design or parallel concurrent downloads. Thus, in the near future, page transfers should tend toward "single ON" downloads.

Let us emphasize that there is a limitation n_{max} on the number of mobiles that can simultaneously be in active transfer phase in the cell.

$$n_{max} = \min(tbf_{max}, N) \tag{2}$$

It involves both the system constraint tbf_{max} and the total mobile population N.

Extensions to finite-length sessions, where each mobile generate ON/OFF traffic during a session and does not generate any traffic during an inter-session, have been provided in [1] and [2]. It is shown in [2] that a very simple transformation of the traffic parameters that consists in increasing the OFF periods by a portion of the inter-session period, enables us to transform the bi-dimensional model developed in [1] into a linear Erlang-like model, with a very good accuracy.

2.3 Markovian analysis

In [5] an analytical model for the performance evaluation of a single-cell GPRS system is derived relying on the traffic model of the previous section. As mentioned above, the strong memoryless assumptions for the ON and OFF distributions are discussed and validated against the most commonly used traffic models. They are shown not only to provide accurate average performance parameters irrespective of the actual distribution but also to provide the most convenient computation framework. Assuming that the likelihood to have more than one event in the elementary time interval of the system (the 20ms radio block duration on GPRS or EDGE) is negligible, the Markovian model is shown to boil down to the linear discrete-time Markov chain given in Fig. 1.

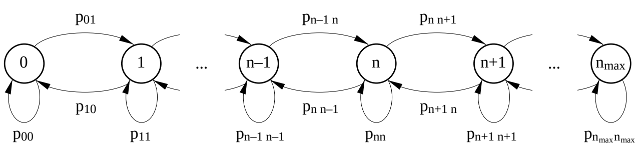


Figure 1: Linear model

The state n of this Markov chain corresponds to the number of mobiles that are simultaneously in active transfer. Let n_0 be the maximum value of n such that $nd < T$:

$$n_0 = \left\lfloor \frac{T}{d} \right\rfloor \tag{3}$$

If there are less than n_0 mobiles in active transfer, the total number of time-slots that can be used in the cell is thus limited to nd, and each mobile receives its maximum downloading bandwidth. Otherwise, all the T time-slots of the cell are used, and each mobile receives a reduced download bandwidth.

The stationary probabilities of having n mobiles in active transfer in the cell can easily be derived from the birth-death structure of this linear Makov chain. By further Taylor series expansions, these probabilities can be expressed as a function of a single dimensionless parameter x as follows (see [5] for details):
for $0 < n \leq n_0$:

$$p(n) = \frac{N!}{n! d^n (N-n)!} x^n p(0) \tag{4}$$

for $n_0 < n \leq n_{max}$:

$$p(n) = \frac{N!}{n_0! d^{n_0} T^{n-n_0} (N-n)!} x^n p(0) \tag{5}$$

where x is given by:

$$x = \frac{t_B \, x_{on}}{x_B \, t_{off}} \tag{6}$$

Note that the product xN characterizes in an aggregate way the traffic load of the cell, as it increases when the size of the downloaded pages (x_{on}) or the number of mobiles (N) increase, and decreases when the reading time of a page (t_{off}) or the coding scheme (x_B) increase. It is in fact the equivalent to Erlang load factor for a finite number of users doing ON/OFF sessions in data traffic.

Finally, $p(0)$ is obtained by normalization:

$$A(n) = \sum_{n=1}^{n_0} \frac{N!}{n! d^n (N-n)!} x^n$$

$$B(n) = \sum_{n=n_0+1}^{n_{max}} \frac{N!}{n_0! d^{n_0} T^{n-n_0} (N-n)!} x^n$$

$$p(0) = \frac{1}{1 + A(n) + B(n)} \tag{7}$$

The performance parameters of the cell can be derived from the stationary probabilities as follows (see [5] for details). The normalized utilization $\tilde{U}$ of the cell, i.e. the mean number of time-slots occupied by GPRS active mobiles, is given by:

$$\tilde{U} = \sum_{n=1}^{n_{max}} p(n) \min(nd, T) \tag{8}$$

The average number $\bar{Q}$ of mobiles in active transfer in the cell is directly obtained as:

$$\bar{Q} = \sum_{n=1}^{n_{max}} n p(n) \tag{9}$$

The normalized throughput $\tilde{X}$, i.e. the average number of time-slots given to a mobile for its transfers is given by:

$$\tilde{X} = \frac{\tilde{U}}{\bar{Q}} \tag{10}$$

Finally, the blocking (or reject) probability P_r, i.e. the probability that a mobile that wants to start the download of a new page cannot do it because the limit of n_{max} mobiles in the cell is reached, is obtained as:

$$P_r = 1 - \frac{1}{x} \frac{\tilde{U}}{\sum_{n=0}^{n_{max}} p(n)(N-n)} \tag{11}$$

Let us note that when $n_{max} = N$ no blocking can occur, and the local balance equations of the Markov chain imply that

$$p(n-1)(N-n+1)\frac{t_B}{t_{off}} = p(n)\min(nd, T)\frac{x_B}{x_{on}}$$

for any $n = 1, ..., n_{max}$, which actually result in having $P_r = 0$.

As a consequence, all the average performance parameters of a single cell can be expressed as a function of the dimensionless parameter x, the cell capacity T, the mobiles capacity d, and the number N of mobiles in the cell.

3. MULTIPLE CELL SYSTEM

3.1 System description

We now assume, as described in Section 1, that traffic may be limited because of a capacity constraint in a network element that controls traffic over a group of P cells. As stated before, this global capacity limit either comes from a software imposed limitation in an equipment (PCU or SGSN module), in order to avoid any capacity overflow, or can been seen as a first approach to model a more complex processing limit constraint. Let M_{max} be the total number of mobiles that can currently be in active transfer in the P cells. Any cell i ($i = 1, ..., P$) subjected to this global constraint may have specific characteristics. Parameters of cell i will be denoted by adding a superscript i to all of its parameters. Assumptions made on each cell i are identical to those made on Section 2. We then define:

- N^i: the total number of mobiles in cell i; Each of these N^i mobiles generates in cell i an ON/OFF traffic having the same characteristics (given by parameters x^i_{on} and t^i_{off}, as described in Section 2.2);
- T^i the total number of time-slots dedicated to GPRS in cell i;
- d^i: the maximum number of time-slots that a mobile can simultaneously use for the downlink traffic in cell i;
- x^i: the dimensionless parameter that both involves the characteristics of the coding scheme and the traffic:

$$x^i = \frac{t_B \, x^i_{on}}{x^i_B \, t^i_{off}} \tag{12}$$

  Note that, each cell may contain mobiles using a different coding scheme (x^i_B) and different traffic parameters (x^i_{on} and t^i_{off});
- $n^i_{max} = \min(tbf^i_{max}, N^i)$: the maximum number of mobiles that can simultaneously be on active transfer in cell i.

Of course, if $\sum_{i=1}^{P} n^i_{max} \leq M_{max}$, the limit does not generate any additional constraint on the system, and each cell can be analyzed using the single-cell model described in Section 2. As a consequence, we only consider here the case where $\sum_{i=1}^{P} n^i_{max} > M_{max}$. In such a system, a mobile in a given cell i will not be able to start a new transfer either because the cell capacity (n^i_{max}) is reached or because the global system capacity (M_{max}) is reached.

3.2 Model description

A direct extension of the single-cell model described in Section 2 would have consisted in developing a multi-dimensional Markov chain, each dimension corresponding to a given cell. Of course such an extension would have resulted in non-manageable complexity when the number P of cells that are subjected to the constraint M_{max} is large. The first objective of this work is to provide a tractable model for any value of P. To fulfill this objective, the idea is to only deal with approximate linear Markov chain models as developed below.

Let us focus on a particular cell i over the P cells of the GPRS/EDGE system. We will denote by "*constrained* cell i", the actual cell i that is subjected to the global capacity limit M_{max}. As explained before, an inactive mobile in constrained cell i may not be able to start a new transfer because of the global capacity limit that implies a coupling with other cells, even if the local cell capacity n^i_{max} is not reached. We then denote by "*unconstrained* cell i" a virtual cell having the same characteristics as cell i but that is not subjected to the overall constraint M_{max}. As a matter of fact, the performance of unconstrained cell i can be derived from the single-cell model of Section 2. The unconstrained steady-state probabilities $p^i_{uc}(n)$ of having n mobiles ($n = 0, ..., n^i_{max}$) in this virtual cell are thus given by relations (4) and (5) by adding a superscript i to all the cell parameters.

The first step of the analysis consists in developing the so-called "aggregate Makov chain" associated with cell i considered. This aggregate model has the same linear structure as the single-cell Markov chain model (Fig. 1), but the transition from any state n to the state $n+1$ is now multiplied by a factor $(1-r^i(n))$ as illustrated in Fig. 2. The objective of the aggregate model is to capture in an aggregate way the effect of coupling between cell i and the other cells of the GPRS/EDGE system, by introducing additional $r^i(n)$ probabilities that decrease the probability of new transfers acceptance. $r^i(n)$ is the probability that an inactive mobile in cell i that wants to start a new transfer cannot do it because the system limit M_{max} is reached, assuming that there are n mobiles currently in active transfer in the cell. As a consequence, $r^i(n)$ is the probability that the system is full when there are n mobiles in the cell considered, and can thus be estimated by the probability that the $P-1$ other cells contain $M_{max} - n$ mobiles. By keeping a linear structure, the aggregate model avoids the problem of complexity of any direct multi-dimensional extension.

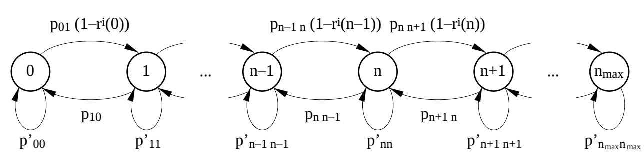


Figure 2: Aggregate model

The challenging task now consists in estimating the probabilities $r^i(n)$ of the aggregate model associated to a given cell i. As stated before, $r^i(n)$ can be estimated as the probability that the set of *constrained* cells $\mathcal{S}^i = \{1, ..., i-1, i+1, ..., P\}$ contains $M_{max} - n$ mobiles in active transfer. In order to derive these probabilities, we first calculate the convolution of the probabilities of the $P-1$ other *unconstrained* cells. The convolution is performed over the steady-state probabilities $p^j_{uc}(n)$ of each unconstrained cell $j \in \mathcal{S}^i$, and enables us to calculate the probability $p^{\mathcal{S}^i}_{uc}(k)$ that the set of unconstrained cells $\mathcal{S}^i$ contains exactly k mobiles, for $k = 1$ to $\sum_{j \neq i} n^j_{max}$ (relation (13)). Note that this step can be performed very efficiently using any convolution algorithm [8].

$$p^{\mathcal{S}^i}_{uc}(k) = \left(\bigotimes_{j \in \mathcal{S}^i} p^j_{uc} \right)(k) \tag{13}$$

$$p^{\mathcal{S}^i}_{uc}(k) = \sum_{\substack{(n^1, ..., n^{i-1}, n^{i+1}, ..., n^P) | \\ \sum_{j \in \mathcal{S}^i} n^j = k \\ n^j \leq n^j_{max} \forall j \neq i}} \left(\prod_{j \in \mathcal{S}^i} p^j_{uc}(n^j) \right)$$

$$\text{for } k = 0, ..., \sum_{j \neq i} n^j_{max}$$

We then propose to estimate the probabilities $r^i(n)$ by normalizing these convoluted probabilities as follows:

$$r^i(n) = \frac{p_{uc}^{\mathcal{S}^i}(M_{max} - n)}{\sum_{k=0}^{M_{max}-n} p_{uc}^{\mathcal{S}^i}(k)} \quad (14)$$

In order to justify the fact that this normalization expression provides a very good estimate for the $r^i(n)$ probabilities, let us first consider the simpler case where $P = 2$, i.e. the global limit M_{max} only involves two cells (1 and 2). If we consider cell 1, $r^1(n)$ is the probability that the constrained cell 2 contains $M_{max} - n$ mobiles in active transfer provided that cell 1 contains n mobiles, i.e. provided that cell 2 cannot contain more than $M_{max} - n$ mobiles in active transfer. In order to derive this probability, we need to develop the model for constrained cell 2 knowing that cell 1 contains n mobiles. As long as there are less than $M_{max} - n$ mobiles in active transfer in cell 2, the global limit M_{max} does not matter for cell 2 that can thus behave as if it was unconstrained. As a consequence the transitions into any state lower than $M_{max} - n$ in the underlying Markov chain are exactly the same as those of the single-cell unconstrained model. This is illustrated in Fig. 3. On the other hand, when cell 2 reaches state $M_{max} - n$, any request for a new transfer is rejected because the global system capacity is reached. The constrained Markov chain is thus truncated to states 0 to $M_{max} - n$ with regard to the unconstrained Markov chain. As a consequence, both Markov chains have the same steady-state frontier equations (restricted to the common states of both chains). The stationary probabilities of the constrained Markov chain can thus be obtained by renormalization over the stationary probabilities of the unconstrained one. This statement justifies the derivation of the $r^1(n)$ probabilities as:

$$r^1(n) = \frac{p_{uc}^2(M_{max} - n)}{\sum_{k=0}^{M_{max}-n} p_{uc}^2(k)} \quad (15)$$

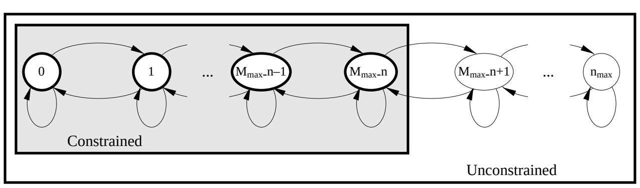


Figure 3: Constrained and Unconstrained models for the remaining cells when $P = 2$

We can easily extend this reasoning to more than 2 cells. If we focus on cell 1, the probability $r^1(n)$, for a given value of n, can be calculated exactly by analyzing the $(P-1)$-dimensional Markov chain modeling the remaining *constrained* system consisting of cells 2 to P, in which the total number of mobiles in active transfer is limited to $M_{max} - n$. Now if we consider the same system without any global constraint, i.e. the set of *unconstrained* cells 2 to P, the resulting Markov-chain would again be a $(P-1)$-dimensional chain, but where each dimension i is allowed to reach the local limit n_{max}^i. The constrained and unconstrained remaining Markov chain models are illustrated in Fig. 4 for the case where $P = 3$ (the remaining system thus contains 2 cells). Now it is easy to understand that as long as the global limit $M_{max} - n$ is not reached in the constrained remaining system, its behavior is exactly the same as that of the unconstrained system. In other words, any steady-state equation of the unconstrained model that only involves states that belong to both models, would also be a steady-state equation for the constrained model. As a consequence, the stationary probabilities of the constrained model can be obtained by renormalization over the stationary probabilities of the unconstrained model. Finally, it is worthwhile noting that, since all cells are independent in the unconstrained model, the stationary probability vector of the underlying Markov chain is simply the product of the marginal probabilities of each unconstrained single-cell Markov chain. This last statement justifies the derivation (14) of the $r^i(n)$ probabilities.

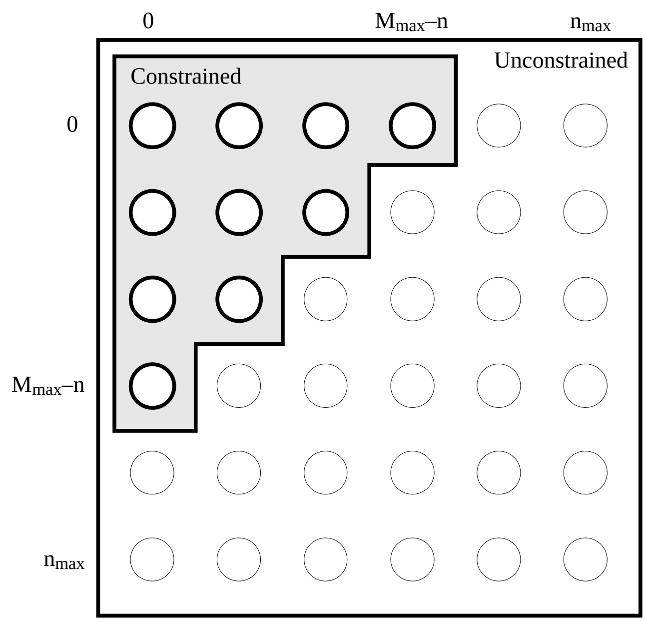


Figure 4: Constrained and unconstrained models for the remaining cells when $P = 3$

We can then inject the $r^i(n)$ parameters in the aggregate model of Fig. 2 and analyze it. The resulting steady-state probabilities of the aggregate model (referenced with a subscript "agg") are thus given by

for $0 < n \leq n_0^i$:

$$p_{agg}^i(n) = \frac{N^i!}{n!(d^i)^n(N^i - n)!}(x^i)^n \cdot \left(\prod_{k=0}^{n-1} \left(1 - r^i(k)\right) \right) p_{agg}^i(0) \quad (16)$$

for $n_0^i < n \leq n_{max}^i$:

$$p_{agg}^i(n) = \frac{N^i!}{n_0^i!(d^i)^{n_0^i} T^{(n-n_0^i)} (N^i - n)!} (x^i)^n \cdot \left(\prod_{k=0}^{n-1} \left(1 - r^i(k)\right) \right) p_{agg}^i(0) \quad (17)$$

Where $p_{agg}^i(0)$ is obtained by normalization.

Finally, we can derive the normalized utilization $\tilde{U}^i$ of any cell i, as well as the average number $\tilde{Q}^i$ of mobiles in active transfer in cell i, and the normalized throughput $\tilde{X}^i$ offered to a mobile for its transfer in cell i, from relations (8), (9) and (10), by replacing the probabilities $p(n)$ by the probabilities $p_{agg}^i(n)$ of the aggregate model obtained from relations 16 and 17.

A special attention must be paid to the blocking probability P_r^i. Indeed, the rejection of a new transfer request in a given cell i can have two different causes in the context of a multiple-cell GPRS/EDGE system: one is related to the local constraint n_{max}^i, the other is related to the global constraint M_{max}. Relation (18) (which is a simple rewriting of single-cell relation (11)) includes both possibilities.

$$P_r^i = 1 - \frac{1}{x^i} \frac{\sum_{n=1}^{n_{max}^i} p_{agg}^i(n) \min(nd^i, T^i)}{\sum_{n=0}^{n_{max}^i} p_{agg}^i(n)(N^i - n)} \quad (18)$$

However, if needed, the blocking probability can be further decomposed into two parts, the "local" blocking and the "global" blocking. Both are evaluated from the aggregate Markov chain associated with cell i. The local blocking is the probability that a new transfer request occurs when cell i already contains n_{max}^i mobiles in active transfer, and can thus be obtained as:

$$P_{r\,local}^i = \frac{p_{agg}^i(n_{max}^i)(N^i - n_{max}^i)}{\sum_{n=0}^{n_{max}^i} p_{agg}^i(n)(N^i - n)} \quad (19)$$

As a consequence, the global blocking probability can easily be derived as the difference between the total blocking and the local blocking:

$$P_{r\,global}^i = P_r^i - P_{r\,local}^i \quad (20)$$

4. MODEL VALIDATION

In this section we validate the analytical model by comparison with simulation results. The simulations have been performed with a simplified event-driven simulator. It assumes the same ON/OFF nature of the traffic and makes the same memoryless assumptions as in our model. However the simulator captures the detailed behavior of the radio resource allocator. Let us remind that these memoryless assumptions have been validated in [4] by comparing them with more realistic assumptions (Pareto and Weibull distributions) using a full end-to-end GPRS chain simulated with OPNET on a few configurations. Opnet includes the full suite of protocol layers, with segmentation, reasembly and relevant overheads, which are not simulated in the simple simulator, but rather replaced by an average increase of the average ON size due to protocol overheads. Our simulator has been validated with Opnet both on single-cell GPRS/EDGE systems and multiple-cell systems. Note finally that, even though the simplified simulator is much faster than the Opnet simulator, it still requires several hours of CPU time to evaluate a single point. Opnet would have been prohibitively slow for us to extensively validate our model.

4.1 Identical cells

First, we consider configurations where the P cells are identical in terms of available radio resources and offered traffic. All the mobiles generate the same traffic (see Table 1 for detailed parameter values). We show the evolution of the normalized utilization $\tilde{U}$, the normalized throughput $\tilde{X}$ and the blocking probability P_r on any given cell, as a function of the number P of cells managed by the central equipment when the capacity limit, given by the maximum number of concurrent transfers M_{max}, is constant. Typically we want to understand the benefit, in terms of overall system capacity, of increasing the number of cells connected to the processing-limited equipment. The results are given in Fig. 5 (a, b, c). We also present in Fig. 5 (d) the stationary probability vector for any cell in the last configuration point, i.e. $P = 20$. Results derived from the analytical model presented in Section 3 and simulations results obtained with the simplified simulator are compared.

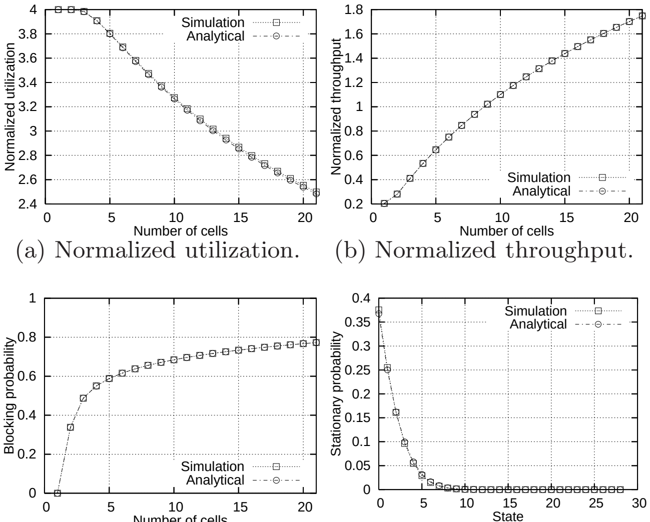
Table 1: Parameter values for each cell.

Parameter	Value	Meaning
T	4	Number of dedicated TS for GPRS
d	4	Maximum number of TS used by a mobile in download
x_B	30 bytes	Payload per radio block (CS2 coding scheme)
N	30	Total number of mobiles in the cell
x_{on}	4000 bytes	Average page size
t_{off}	7s	Average reading time
M_{max}	30	Global capacity limit

As can be seen on the figures, the curves corresponding to analytical results and those corresponding to simulation are almost superimposed. The maximum error is calculated as the relative difference between the simulation and the analytical results on each performance parameter for each cell. The maximum error never exceeds 0.5% for any performance parameter in any configuration. The 20 different configurations (corresponding to P varying from 1 to 20) have been obtained in more than 40 hours with the simulation tool and instantaneously with the analytical model. The model captures very precisely the behaviour of the simulated system.

4.2 Different cells

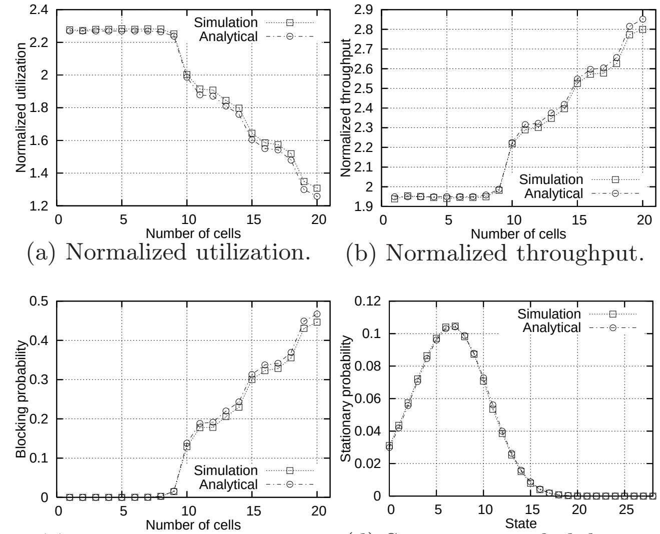
We now drop the assumption of identical cells to further validate the analytical model and to capture more realistic situations where traffic is not spatially homogeneous. The characteristics of all cells in terms of offered traffic and radio conditions are randomly generated (see Table 2 for detailed values). Each cell contains a finite population of homogeneous GPRS mobiles, i.e. we assume that all the mobiles in



(a) Normalized utilization. (b) Normalized throughput.

(c) Blocking probability. (d) Stationary probabilities.

Figure 5: Comparison with simulation results for identical cells.



(a) Normalized utilization. (b) Normalized throughput.

(c) Blocking probability. (d) Stationary probabilities.

Figure 6: Comparison with simulation results for different cells.

a given cell have the same traffic characteristics and use the same effective coding scheme. Typically, we could represent a cell with a majority of business users, having a specific call profile, and with given average radio conditions captured in the effective coding scheme. Another cell would have more consumer traffic and a different radio quality.

We focus on the first cell of the system and study the evolution of its performance when the total number P of cells managed by the equipment varies from 1 to 20. The introduction of a new cell in the system influences the first cell's performance. It directly depends on the traffic load introduced by the new cell and explains the step behavior of the performance curves. Fig. 6 (a, b, c) show the evolution of the normalized utilization $\tilde{U}$, the normalized throughput $\tilde{X}$ and the blocking probability P_r of the first cell. The stationary probabilities of the first cell for the last point, i.e. when $P = 20$, are given in Fig. 6 (d). Again, analytical results and simulations are compared in all figures.

Table 2: Different cells parameter ranges.

Parameter	Range	Meaning
N	[5-50]	Total number of mobiles in the cell
x_B	[20-36] (bytes)	Payload per radio block
x_{on}	[1000-64000] (bytes)	Average page size
t_{off}	[6-60] (s)	Average reading time

Once again, the maximum relative error is estimated for all performance parameters, in all cells, and for all configurations. We notice that the maximum error is about 7% on the blocking probability parameter. Moreover, this gap only happens in ranges over 20% of blocking which is beyond the scope of common studies for dimensioning of real GPRS/EDGE networks. These results show that a very good agreement is obtained between the analytical model and simulation. Finally, the same conclusions on CPU-time can be made (several days for the simulation results and instantaneous for the analytical results).

5. PERFORMANCE RESULTS

In this section, we use the analytical model developed in Section 3 to derive performance curves. We assume that all the cells are identical as our purpose is to derive a first set of general conclusions on the system behaviour. Obviously, similar studies can be performed on heterogeneous cells systems if needed with no additional complexity. We cover an extensive domain, going beyond the normal working range of a commercial GPRS/EDGE network, with blocking values going higher than maximum acceptable values. Thus we are able to bring out asymptotic behavior, quite useful to derive simpler models. More realistic applications of our analytical model will be given in the subsequent section.

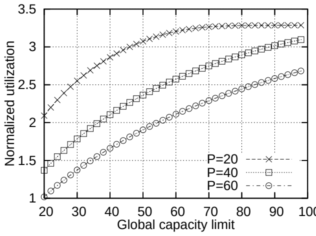
All mobiles are assumed to use 4 time-slots in the downlink ($d = 4$) and to generate the same traffic load: the average size x_{on} of a downloaded page is 4000 bytes and the average reading time t_{off} is 10 s. The same GPRS coding scheme is used (CS2) with good radio conditions: $x_B = 30$ bytes. Note that these values lead to $x \approx 0.267$.

The influence of the main input parameters is studied on the following performance parameters: the normalized radio utilization $\tilde{U}$, the average number $\bar{Q}$ of mobiles in active transfer in any cell, as well as the normalized throughput $\tilde{X}$ and the blocking probability P_r of any mobile in the system.

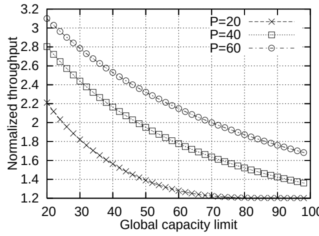
5.1 Influence of the global capacity limit

First, we study the influence of the global capacity limit M_{max}. The number of cells P is set to 20, 40 and 60, and M_{max} is varied. Not surprisingly, and as shown in Fig. 7 (d), the blocking probability P_r decreases as M_{max} increases. Then there are more mobiles in active transfer sharing the radio resource of the cell. Thus when M_{max} increases, the radio utilization of the cell increases and the throughput offer to each mobile decreases. Conversely, we can say that the central capacity limit M_{max} acts as an admission control scheme: when less mobiles are allowed in the system, they are given a better throughput.

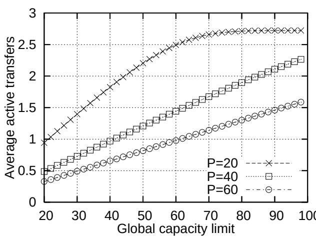
If the system is saturated, $\bar{Q}$ (Fig. 7 (c)) has obviously a linear behavior: the sum of active mobiles over the P cells is almost equal to the global capacity limit M_{max} ($\bar{Q} \approx \frac{M_{max}}{P}$). Note that none of the other performance parameters show this linear behavior even when the system is saturated.



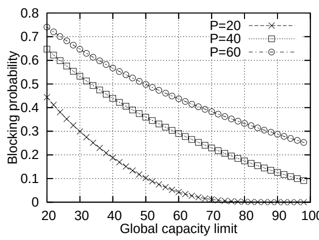
(a) Normalized utilization.



(b) Normalized throughput.



(c) Average active transfers.



(d) Blocking probability.

Figure 7: Cells performance when M_{max} varies.

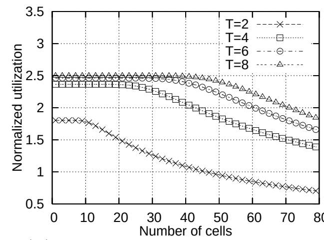
5.2 Influence of the number of dedicated time-slots per cell

We now focus on the influence of T, the number of dedicated time-slots for GPRS/EDGE in the TDMA. M_{max} is set to 40 and we draw the performance curves for different values of T.

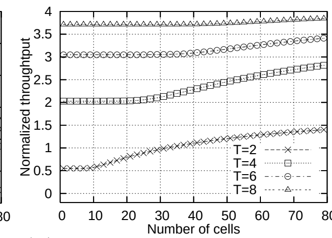
5.2.1 Varying the number of cells

We now study the influence of P, the total number of cells constrained by the global capacity limit M_{max}. As previously mentioned, a typical application is to assess the benefit of increasing the cell connectivity of the centralized processing-limited equipment managing the group of cells. As shown in Fig. 8, the performance curves are made of two parts. For low P values, the likelihood to have M_{max} concurrent transfers is small: each cell behaves as if it were alone. Performance curves are constant and the single-cell model described in Section 2 applies. For large P values, the global capacity limit is felt on each cell. The total number of active transfers in the system grows towards M_{max}, hence the average number of active transfers per cell decreases as $1/P$. As a consequence, the blocking probability increases (the global blocking $P^i_{r\,global}$ of any cell i becoming the main contributor to the total blocking probability P^i_r), but the throughput increases and the radio utilization decreases: a mobile has less chance of being accepted, but once it is, the likelihood that another mobile on the same cell is doing a simultaneous transfer is becoming small, hence its granted throughput is larger.

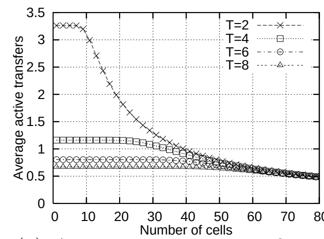
Higher values of T postpone the global limitation effect and have a strong benefit on system performance. When T



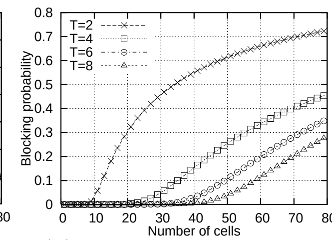
(a) Normalized utilization.



(b) Normalized throughput.



(c) Average active transfers.



(d) Blocking probability.

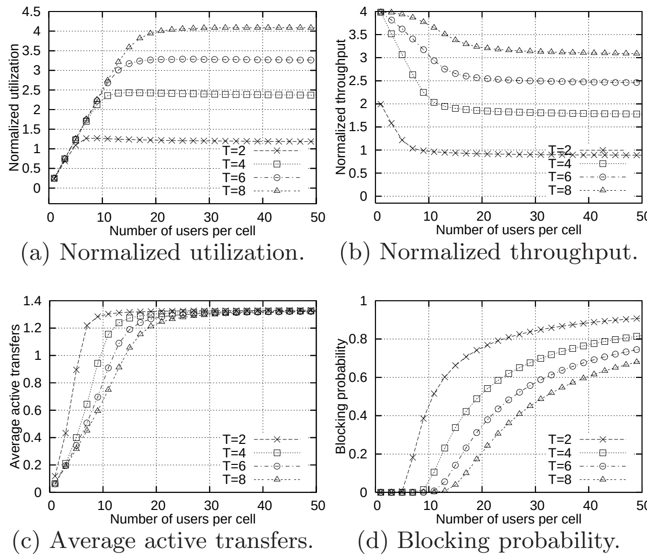
Figure 8: Cells performance when P varies for different values of T.

is high, mobiles can obtain more time-slots as long as their downloading capacity d allows it. They can download their pages faster and leave the system earlier. For a blocking rate of 2%, for $T = 2$, no more than 10 cells can cope with the global capacity constraint. With $T = 4$ (resp. $T = 8$) 30 (resp. 50) cells are allowed. Too small T values have a bad effect not only on throughput but, more importantly, on blocking.

5.2.2 Varying the load per cell

We now investigate the influence of T as the traffic load in each cell varies. As shown in Section 2.3, the traffic load is an increasing function of both parameters x and N. In this section, the traffic load is varied only by varying N, the number of active GPRS mobiles in the cell (we have obtained similar results when x varies and N is constant but they are not shown here)

As previously, performance curves (Fig. 9 (a, b, c)) are made of two distinct parts, the low traffic limit and the saturated system. In the former, the system is similar to the single cell case. Its performance can thus be obtained from the single-cell model presented in Section 2 (relations (8) to (11)). Radio usage grows linearly with the number of users, and throughput decreases accordingly. In the latter, the only element that appears to have a significant evolution with offered load is the blocking rate (Fig. 9 (d)). The transition between the two regimes occurs when the number of simultaneous transfers is about 90% of the system limit. We have studied extensively these asymptotic behavior in [18] and their application to ease the performance evaluation process.. When the number of users increases, it essentially results in more transfer requests which are rejected. As in the previous case, increasing T is very beneficial, as it dereases the transfer duration and it decreases the blocking probability. Note in passing that in a real-world dimensioning exercise, increasing T has a cost, which is an increase in transport and connectivity requirements for the data traffic. This cost will have to be balanced against the corresponding benefits.



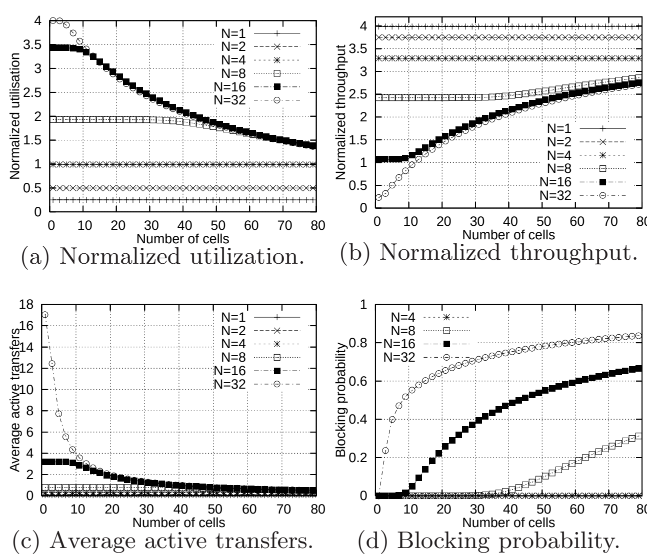
(a) Normalized utilization. (b) Normalized throughput.

(c) Average active transfers. (d) Blocking probability.

Figure 9: Cells performance when N varies for different values T.

5.3 Influence of the traffic load per cell

In this section, we study the effect of the traffic load per cell when P varies. Curves shown in Fig. 10 (a, b, c) are quite similar to those of Section 5.2.1. We can again see the two single cell and coupled cells regimes. This has also been discussed in [18] and led us to develop asymptotic expressions for these two regimes.



(a) Normalized utilization. (b) Normalized throughput.

(c) Average active transfers. (d) Blocking probability.

Figure 10: Cells performance when P varies for different traffic loads.

5.4 Performance graphs

This section provides some examples of graphs that can be instantaneously obtained with our analytical solution and that can't even be thought of with simulators because of their prohibitive computation time. We draw 3-dimensional performance surfaces where performance parameters can be drawn as a function of e.g. N and x. For each performance parameter, the surface is cut out into level lines and the resulting 2-dimensional projections are drawn. The step between level lines can be arbitrarily chosen as a function of the required precision. The normalized radio resource utilization of any given cell $\tilde{U}$, the normalized throughput $\tilde{X}$ and the blocking probability P_r for any mobile in the system are presented in Fig. 11, 12 and 13 for $P = 30$ and $M_{max} = 40$. Each graph is the result of several thousands of input parameter sets. Obviously, a simulation tool or even any multi-dimensional Markov chain requiring numerical resolution would have precluded the drawing of such graphs. They allow to directly derive any performance parameter knowing the traffic load profile. It is described by the couple (N, x) (remember that we assume that all cells are identical). Such parameters can be measured on real systems. They depend on the used applications (WAP, web, mail, etc.).

These performance graphs can be used as easily and efficiently as the classical Erlang graphs used for the dimensioning of circuit traffic.

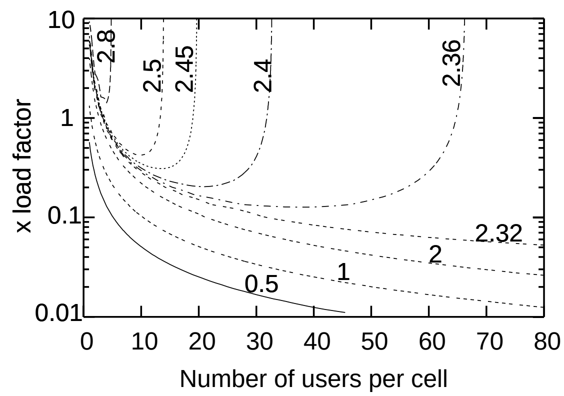


Figure 11: Normalized utilization for any traffic load profile.

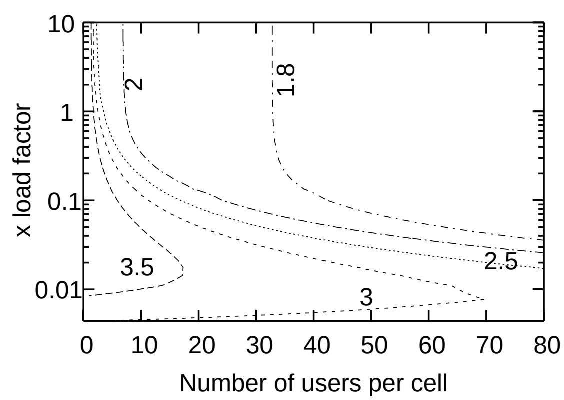


Figure 12: Normalized throughput for any traffic load profile.

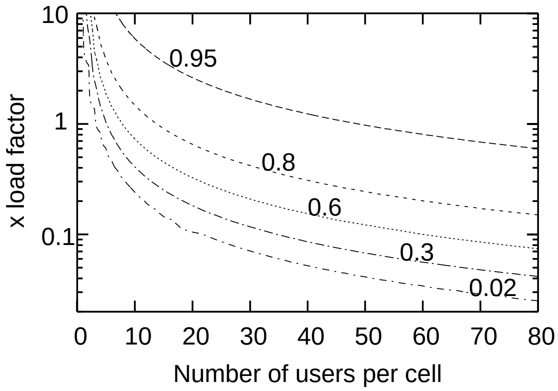


Figure 13: Blocking probability for any traffic load profile.

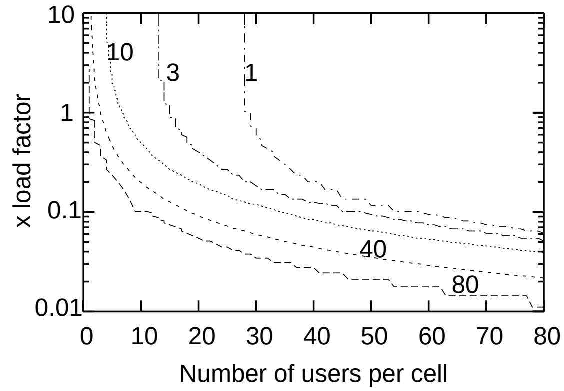


Figure 14: A dimensioning graph for finding an optimal P with $P_r \leq 2\%$

6. DIMENSIONING STUDY

In this section, we present how our model can be applied in dimensioning studies. Once again, our modeling framework allows very fast computations, which in turn allow complex iterative dimensioning analyses.

6.1 Maximum blocking probability

We assume here that the network dimensioning is based on a maximum acceptable blocking rate of 2% for data transfer requests. This value is similar to the voice blocking value generally used. In GPRS or EDGE, a transfer request rejection results in 5 seconds idle time before a subsequent request is allowed. Hence it has a strong impact on end-user quality of experience. For this target blocking rate, we want to find the values of:

- P_{max}: the maximum number of cells;
- N_{max}: the maximum number of GPRS mobiles that can be admitted in each cells.

The method used in Section 5.3 can also be used to derive the maximum number of cells the system can have to guarantee the requested QoS for a given traffic load. In Fig. 14, we plot a set of curves for a 2% blocking probability. For a given traffic load, the point of coordinates (x, N) in the graph gives the optimal P value, by interpolation between the two surrounding curves.

In the same way, the graph in Fig. 15 gives the maximum value of the number N of mobiles in the cell to guarantee a 2% blocking probability. The same method is applied. The intersection point for a chosen P and x is located between two level lines and the level line with the lower value gives N_{max}, the maximum number of GPRS mobiles the cells can contain to guarantee the QoS criterion.

6.2 Minimum normalized throughput

The normalized offered throughput is another possible QoS requirement. The network dimensioning is then based on a minimum average throughput obtained by each GPRS mobile for its transfers. A typical 1 time-slot threshold is chosen, i.e. a mobile that starts downloading a page has the guarantee to obtain at least 1 time-slot per TDMA for the entire transfer duration. For a GPRS CS2 coding scheme, it corresponds to a minimum throughput of 1500 bytes/s.

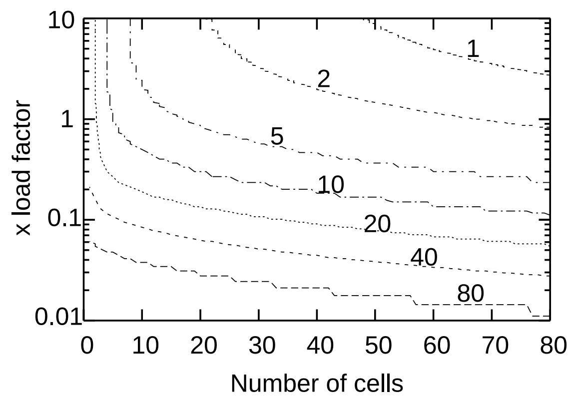


Figure 15: A dimensioning graph for finding an optimal N with $P_r \leq 2\%$

As explained in Section 5, the normalized throughput is an increasing function of P. We then have to find the minimum value P_{min} of cells to guarantee the throughput threshold. In Fig. 16, the intersection point for a couple (x, N) is located between two level lines and the line with the lower value gives P_{min}.

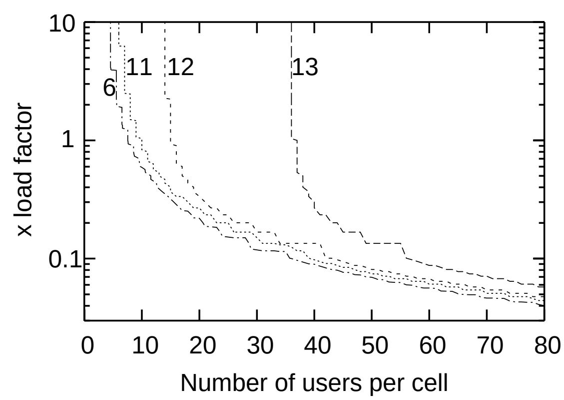


Figure 16: A dimensioning graph for finding an optimal P with $\tilde{X} \geq 1TS$

Fig. 17 finally shows the dimensioning curves that give the maximum value N_{max} of GPRS users per cell to guarantee the throughput QoS criterion.

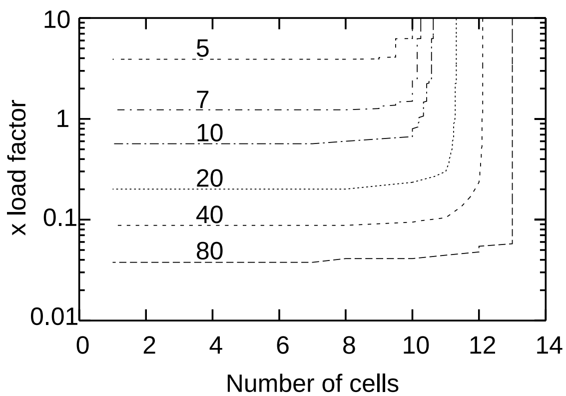


Figure 17: A dimensioning graph for finding an optimal N with $\tilde{X} \geq 1TS$

7. CONCLUSION

We have been able to provide a computationally simple model of the a priori complex system made of a group of cells in a cellular network coupled by capacity limitation in a centralized equipment handling packet traffic. We have demonstrated that a multi-dimensional Markovian model, which would be a natural generalization of a single-cell model to multiple-cell systems, can be decomposed into P aggregate linear Markov chains. We have shown that the performance parameters of a given cell in the aggregate model can easily be derived from the performance parameters of the other cells when they are not constrained, which can therefore be solved by single-cell analytical models.

Our aggregate analytical model provides the expected computational efficiency and accuracy necessary for complex performance and dimensioning analyses. As an example of application we have studied a dimensioning problem with different realistic QoS guarantees. Similar studies, not presented here, have been used to quantify limitations in the design of GPRS/EDGE network components and drive design evolution. We have now a modeling framework allowing us to model not only a single cell but a full group of cells coupled by a central dimensioning constraint. Its ability to perform very fast computation allows not only to dimension networks based on traffic load asumptions but also to use it as the forward modeling step in an iterative dimensioning process. We have given such examples where reference curves provide the system parameter values that are needed to reach required quality figures. Other examples such as a network dimensioning where the generated traffic induces interference which in turn decrease throughput could as well be addressed.

In the future we plan to investigate more complex central capacity limitations and their impact on the GPRS/EDGE network performance. We plan to fully extend the validation work with real-world traffic traces. We also intend to extend this work and methodology to UMTS and HSDPA modeling. In these wideband CDMA networks, all the interactions due to radio resource management induce significantly more complexity. Having a computationally fast module for packet-based traffic modeling is an essential asset.

7.1 Acknowledgments

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8. REFERENCES

[1] B. Baynat, K. Boussetta, P. Eisenmann, and N. Ben Rached. A discrete-time Markovian model for GPRS/EDGE radio engineering with finite-length sessions traffic. In *International Symposium on Performance Evaluation of Computer and Telecommunication Systems (SPECTS'04)*, May 2004.

[2] B. Baynat, K. Boussetta, P. Eisenmann, and N. Ben Rached. Towards an Erlang-Like formula for the performance evaluation of GPRS/EDGE networks with finite-length sessions. In *3rd IFIP-TC6 Networking Conference*, May 2004.

[3] B. Baynat, K. Boussetta, P. Eisenmann, and S. Trabelsi. Extended Erlang-B law for performance evaluation of voice/data policies in GSM/(E)GPRS Networks. In *Proc of IEEE PIMRC*, September 2005.

[4] B. Baynat and P. Eisenmann. An Erlang-Like formula for GPRS/EDGE network engineering. In *Technical report.* Nortel Networks - LIP6, 2003.

[5] B. Baynat and P. Eisenmann. Towards an Erlang-Like formula for GPRS/EDGE network engineering. In *IEEE International Conference on Communications (ICC)*, June 2004.

[6] S. Ben Fredj, T. Bonald, Proutiere, G. A. Regnie, and J. Roberts. Statistical bandwidth sharing: A Study of congestion at flow level. In *ACM Sigcomm*, 2001.

[7] A. Berger and Y. Kogan. Dimensioning bandwidth for elastic traffic in high-speed data networks. In *IEEE/ACM Trans. Networks*, pages 643–654, October 2000.

[8] S. Bruel and G. Balbo. Computational Algorithms for Closed Queuing Networks, Operating and programming systems series. P.J. Denning ed., New York, Oxford: North-Holland, 1980.

[9] J. Cai and D. Goodman. General Packet Radio Service in GSM. In *IEEE Comm. Magazine*, pages 122–131, October 1997.

[10] Y. Chung, D. Sung, and H. Aghvami. Steady State Analysis of Mobile Station State Transitions for General Packet Radio Service. In *Proc of IEEE PIMRC*, September 2002.

[11] Fang and D. Ghosal. Performance Modeling and QoS Evaluation of MAC/RLC Layer in GSM/GPRS Networks. In *Proc. of IEEE International Conference on Communications*, May 2003.

[12] C. H. Foh, B. Meini, B. Wydrowski, and M. Zukerman. Modeling and Performance Evaluation of GPRS. In *Proc. of IEEE VTC*, May 2001.

[13] D. Heyman, T. Lakshman, and A. Neidhardt. New method for analyzing feedback protocols with applications to engineering web traffic over the internet. In *Proc. of the ACM Sigmetrics*, 1997.

[14] C. Lindemann and A. Thummler. Performance Analysis of the General Packet Radio Service. In *Computer Networks*, pages 1–17, January 2003.

[15] M. Mahdavi, R. Edwards, and P. Ivey. Performance Evaluation of a Data Subsystem in GSM/GPRS Using

Complete Sharing of Available Bandwidth. In *London Communications Symposium*, 2001.

[16] M. A. Marsan, P. Laface, and M. M. Packet Delay Analysis in GPRS Systems. In *IEEE INFOCOM*, 2003.

[17] S. Ni and S. Haggman. GPRS performance estimation in GSM voice and GPRS shared resource system. In *Proc. of IEEE Wireless Communication and Networking Conference(WCNC)*, pages 1417–1421, 1999.

[18] G. Nogueira, B. Baynat, and P. Eisenmann. Asymptotic behavior of a GPRS/EDGE network with several cells controlled by a global capacity limit. In XIth *International Symposium on Applied Stochastic Models and Data Analysis (ASMDA)*, May 2005.

[19] M. Oliver and C. Ferrer. Overview and Capacity of the GPRS (General Packet Radio Service). In *Proc. of PIMRC'98*, 1998.

[20] S. Pedraza, J. Romero, and J. Muoz. (E)GPRS Hardware Dimensioning Rules with Minimum Quality Criteria. In *Vehicular Technology Conference*, pages 391–395, 2002.

[21] P. Stuckmann and F. Muller. GPRS Radio network capacity considering coexisting circuit switched traffic sources. In *European conference on wireless technologie*, 2000.

[22] P. Stuckmann and O. Paul. Dimensioning Rules for GSM/GPRS Networks. In *Proc.of the Aachen Symposium on Signal Theory*, pages 169–174, 2001.

[23] U. Vornefeld. Analytical Performance Evaluation of Mobile Internet Access via GPRS Networks. In *European Wireless*, February 2002.