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Formalization and Strictness of Simulation Event Orderings

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Abstract

This paper advocates the use of a formal framework for analyzing simulation performance. Simulation performance is characterized based on the three simulation development process boundaries: physical system, simulation model, and simulator implementation. Firstly, we formalize simulation event ordering using partially ordered set theory. A simulator implements a simulation event ordering, and incurs implementation overheads when enforcing event ordering at runtime. Secondly, we apply our formalism to extract and formalize the simulation event orderings of both sequential and parallel simulations. Thirdly, we propose the relation stricter and a measure called strictness for comparing and quantifying the degree of event dependency of simulation event orderings respectively.

1. Introduction

As the size and complexity of simulations grow, the computational demand required is fast becoming a limited factor in solving large and complex real-world problems. Consequently, understanding simulation performance becomes increasingly important. Parallel simulation speeds up simulation execution by distributing the simulation across a number of processors. In parallel simulation, a physical system is viewed as a number of physical processes that interacts in some fashion [11]. In the Virtual Time simulation modeling paradigm, each physical process is modeled by a logical process (LP) [12]. The interactions between physical processes are modeled by exchanging timestamped events between the corresponding logical processes. Parallelism is exploited by simulating LPs concurrently.

Research in parallel simulation in the last decade has resulted in a number of synchronization protocols [11]. These protocols, introduced in an algorithm fashion, are frequently evaluated by comparing its performance among protocols [11]. Moreover, the performance metrics and benchmarks used vary among the different studies. A serious drawback is the lack of performance comparison framework. We proposed a time and space performance evaluation framework based on the concept of event ordering [17][20]. Event ordering in simulation refers to a set of rules that is used to order a set of events. The framework characterizes simulation performance along the three natural boundaries in simulation modeling and analysis (see Table 1) [17]. The physical system layer corresponds to real world systems, the simulation model layer corresponds to different simulation event orderings that can be used to simulate a real world system, and the simulator layer corresponds to the simulator implemented to enforce a simulation event ordering. The layered approach provides a framework to study the factors affecting simulation performance from the physical system to its simulator implementation.

Layers	Types of Event	Event Ordering
Physical System	Real events	One
Simulation Model	Real events	Many
Simulator	Real events + Overhead	One or more implementations for a given event ordering

Table 1. Layered Simulation Performance Framework

Event ordering (or message ordering) has been studied in the time management component of High Level Architecture (HLA) [11]. The simulation of a physical system in HLA is distributed across a number of federates. Message ordering in HLA time management dictates the ordering of messages within each federate. Fujimoto et al. introduces five message orderings which form a spectrum of orderings where at one extreme, messages are not ordered; and at the other extreme, messages are totally ordered based on their timestamps [9]. To exploit the temporal uncertainty in a simulation model, Fujimoto proposes Approximate Time (AT) and Approximate Time Causal (ATC) orders [10]. Recently, Zhou et al. investigate the causality issue in distributed simulation and propose the causal receive ordering [23]. In parallel simulation, event ordering dictates the ordering of events within each LP and across LPs. To produce correct simulation result, events in the same LP are executed in non decreasing timestamp order [11]. This constraint is referred to as local causality constraint (*lcc*).

This paper discusses the formalization of simulation event orderings based on partially ordered set (poset). This formalization provides a theoretical foundation for carrying out performance analysis of simulation. If events with the same timestamp are grouped as a set or there is a priority function that can differentiate those events, there will only be one event ordering in a physical system. Simulation can use different event orderings to exploit parallelism in the physical system. This paper shows that a simulator implements a specific event ordering. Two major benefits may be derived from the separation of simulation event ordering from its implementation. First, this facilitates the understanding of the relationship of different event orderings. We propose the relation *stricter* to compare different event orderings. Second, the performance of different event orderings can be evaluated independent of implementation overheads [17][20]. The separation between event ordering and its implementation is motivated by research in memory operation orderings in memory consistency model [6] and message ordering in broadcast services in distributed system [1]. Our work is different from Critical Path Analysis (CPA) that is also used to analyze the performance of parallel simulation [2]. CPA uses an event dependency graph that is based on *happened before* event ordering [13]. Hence, CPA is a subset of our event ordering analysis.

The rest of this paper is organized as follows. Section 2 formalizes the concept of simulation event ordering. We apply this formalism to extract and formalize simulation event orderings in both sequential and parallel simulation. In section 3, we propose the stricter relation and apply this concept to analyze a number of event orderings. We show the empirical result in section 4. Our concluding remark is in section 5.

2. Formalization of Event Orderings

In this section, we propose to formalize simulation event ordering based on partially ordered set (poset). Research in poset theory was triggered by Dushnik and Miller's publication in 1941 [7]. They propose the definition of partial order as given in Definition 1.

Definition 1. *An order R over S (where S is a set) is called a* ***partial order*** *if R is anti-reflexive (i.e.* $(x, x) \notin R$*), anti-symmetric (i.e. either* $(x, y) \in R$ *or* $(y, x) \in R$*), and transitive.*

For example, an order "descendant of" for a given set of people is of partial order. However, an order "friend of" for a given set of people may not be a partial order depending on the given set of people. This leads to the concept of *partially ordered set* [7].

Definition 2. *A* ***partially ordered set (poset)*** *is a tuple (S, R) where S is a set and R is a partial order on the set S.*

Definition 3. *A* ***simulation event ordering*** *(or event ordering in short) is a tuple* (E, S_R) *where E is a set of events and* S_R *is a set of comparable events based on event order R. Event order R must be anti-reflexive, anti-symmetric and transitive.*

Based on the definition of poset, we formalize event ordering in Definition 3. Just as a poset that has two components, an event ordering also comprises two main components: a set of events *E* and an event order *R*. An event order *R* refers to a set of rules that is used to order events. A pair of events $(x, y) \in S_R$ denotes event *x* is ordered before event *y* in event order *R*. Two events *x* and *y* are *comparable* if either $(x, y) \in S_R$ or $(y, x) \in S_R$, otherwise *x* and *y* are non-comparable (or *concurrent)*.

2.1 Physical System

An event order in the physical system corresponds to how events in the physical system are ordered. Based on the physical time, there is only one event order for any physical system, i.e. an event with a smaller physical time is ordered before an event with a larger physical time (Definition 4). The definition of predecessor and antecedent (Definition 5 and 6) will be used throughout this paper.

Definition 4. *Let x be an event in a physical system and x.ts the physical time when event x happens. The event order in any physical system dictates that for all x and y (where* $x \neq y$*), x is ordered before y if and only if* $x.ts < y.ts$.

Definition 5. *Event x is the* ***predecessor*** *of y (denoted by y.pred = x), if x and y are at the same service center and there is no other event z that is also at the same service center such that* $x.ts < z.ts < y.ts$.

Definition 6. *Event x is the* ***antecedent*** *of y (denoted by y.ante = x), if x spawns y.*

2.2 Simulation Model

Based on the virtual time paradigm, a simulation model emulates a physical system and the interaction among physical processes in the physical system (see Figure 1). Each physical process in the physical system is mapped onto a logical process (LP) in the simulation model. Each event in the simulation model models an event in the physical system. The simulation time of an event in the simulation model models the physical time of the corresponding event in the physical system. The event ordering in a physical system can be modeled and simulated using different event orderings to exploit different degrees of event parallelism.

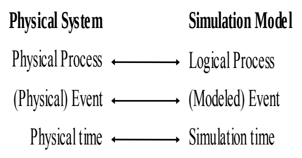


Figure 1. Physical System and Simulation Model

Lamport defined happened before partial order and total order [13]. He proved that both orders are anti-reflexive, anti-symmetric and transitive which match our definition of simulation event order (Definition 3). Hence, we refer to these event orders as partial event order and total event order respectively (see Definition 7 and 8). The priority function in total event order is used to decide which event should be processed when two or more events have the same timestamp. Based on interval order in poset [16], we formalize timestamp event order and time-interval event order [17][20].

Definition 7. ***Partial event order*** *imposes that event x is ordered before event y in if (y.pred = x) or (y.ante = x).*

Definition 8. ***Total event order*** *imposes that event x is ordered before event y in iff (x.ts < y.ts) or (x.ts = y.ts and priority(x) < priority(y)).*

Definition 9. ***Timestamp event ordering*** *imposes that event x is ordered before event y in iff x.ts < y.ts.*

Definition 10. ***Time-interval event ordering*** *imposes that event x is ordered before event y in if: (y.pred = x) or (y.ante = x) or (x.ts + W < y.ts), where W is a constant window size.*

2.3 Simulator

A simulator, written as a sequential program or a parallel program, is an implementation of a simulation model. In parallel simulation, a synchronization algorithm (or simulation protocol) is required for maintaining correct event ordering across processors. Enforcing event ordering at runtime incurs implementation overhead such as null messages in CMB protocol and rollback in TW protocol that results in performance loss.

To show that each simulator implements a certain event order, we extract and formalize the ordering rules of a number of simulator implementations. These include sequential simulation and parallel simulation protocols such as CMB [5], Bounded Lag [14], Time Warp [12] and Bounded Time Warp [21].

2.3.1 Sequential Simulation

The sequential simulation algorithm is presented in Figure 2. Events in sequential simulation are totally ordered (only one event is executed at any time). To enforce this ordering, sequential simulation maintains a future event list (FEL) where events are sorted in chronological timestamp order. In line 2, the function *f* returns the event with the smallest timestamp in the future event list. FEL enables sequential simulation to execute an event with the smallest timestamp (line 12). In case of a tie (i.e. $M \neq \emptyset$ in line 14), an event with the highest priority will be chosen (*z* in line 15). Issues and examples on implementing the priority function have been studied in [18][22]. Lemma 1 formalizes the event ordering in sequential simulation.

```
SEQUENTIAL SIMULATION
1.  initialize
2.  while (~stop) {
3.    e ← f(FEL)
4.    local_clock ← e.ts
5.    FEL ← FEL – {e}
6.    E ← execute (e)
7.    FEL ← FEL ∪ E
8.    stop ← g()
9.  }
10.
11. f(L):event {
12.   x ← head(L)
13.   M ← {y | ∀y∈L • y.ts = x.ts}
14.   if (M = ∅) return x
15.   else return {z | ∀y∈M ∃!z∈M y≠z •
            priority(z)>priority(y)}
16. }
```

Figure 2. Algorithm of Sequential Simulation

Lemma 1. *Sequential simulation implements total event order.*

Proof. Sequential simulation employs a global event list that is sorted by the smallest timestamp first. This guarantees that event *x* is ordered before event *y* if and only if *x.ts* < *y.ts*. The use of a priority function when more than one event have the smallest timestamp guarantees that if *x.ts* = *y.ts* event *x* is ordered before event *y* if and only if *priority(x)* < *priority(y)*. ❑

2.3.2 CMB Protocol

The algorithm of CMB protocol [5] is given in Figure 3. Each LP maintains a list of LPs that may send events to it (for LP *x*, it is denoted by *SENDER*(*x*)). The ordering rule of CMB protocol imposes that only safe events are executed. An event in LP *x* is safe for execution if no other LP ∈ *SENDER*(*x*) will send any event with a smaller timestamp to LP *x*. Therefore, to maintain this ordering, LP *x* must wait to receive events from all other LPs (see line 4). This could lead to deadlock as all LPs are blocked. To avoid deadlock, null messages are introduced. Each null message is stamped with a timestamp *ts* which equal to LP's local simulation clock plus a lookahead value (line 12) to indicate that the sending LP will never transmit any events with a smaller timestamp than *ts*.

Each LP maintains an event list (EL), a set of input buffers (IB) and a set of output buffers (OB). IB[*i*] of an LP *x* stores the incoming message from $LP_i \in$ *SENDER*(*x*). OB[*i*] stores the messages that will schedule events in LP_i. An LP is blocked if at least one of its IBs is empty (line 4). In lines 5-6, an event with the smallest timestamp is chosen from its IBs and EL for execution. Line 7 removes the chosen event from the corresponding list (one of the IBs or EL). The local clock is updated in line 8. In line 9, an event execution may schedule a set of *internal events* (IE) and a set of *external events* (EE). The internal events (i.e. scheduled to happen in the same LP) are saved to EL (line 10) and external events (i.e. scheduled to happen in other LPs) are saved to their respective OB (line 11). Line 12 sets a null message with a timestamp equal to the local clock plus a lookahead value. Line 13 adds a null message to any empty OB. Line 14 sends all the external events and null messages in OBs. Finally line 16 checks the stopping condition.

```
CMB PROTOCOL
1. initialization
2. run all LPs

LOGICAL PROCESS
3. while (~stop) {
4.     while (∃i IB[i] = ∅) {}
5.     L ← EL ∪ {∀i IB[i]}
6.     e ← f(L)
7.     if (∃i e∈IB[i]) IB[i] ← IB[i]-{e}
       else EL ← EL-{e}
8.     local_clock ← e.ts
9.    {IE, EE} ← execute (e)
10.    EL ← EL ∪ IE
11.    ∀i OB[i] ← OB[i] ∪ {z|z∈EE • z.lp=i}
12.    nullMsg.ts ← local_clock + lookahead
13.    ∀i if (OB[i] = ∅) OB[i] ← OB[i] ∪
           {nullMsg}
14.    ∀i send (OB[i])
15.    stop ← g()
16.}
```

Figure 3. Algorithm of CMB Protocol

Lemma 2. *CMB protocol implements an event order whereby event x is ordered before event y if:*

1. *y.pred = x, or*
2. *x.ts + la < y.ts.*

Proof. The conditional statement in line 4 (Figure 3) ensures that an LP has to wait until all LPs in its SENDER list have sent their events. This ensures that an LP always executes events scheduled in it in timestamp order. Hence, for all events in the same LP, if *y.pred = x* then *x* is ordered before *y*. Further, event *y* in LP_j is executed only if it has the smallest timestamp among the unprocessed events of all LP ∈ *SENDER*(LP_j). Therefore, event *x* in any LP ∈ *SENDER*(LP_j) is ordered before event *y* only if *x.ts + la < y.ts* where *la* is the lookahead value. ❑

Researchers have proposed various optimizations such as demand driven protocol [3], and carrier null message protocol [4] to reduce the null-message overhead,. These optimizations do not alter the event ordering in the original CMB protocol, but rather, they can be seen as different implementations of the same event order.

2.3.3 Bounded Lag Protocol

Lubachevsky proposed the Bounded Lag (BL) protocol which combines two main rules: bounded lag restriction and minimum propagation delay [14]. Bounded lag restriction imposes that events can be executed concurrently if they are within the same time window. Minimum propagation delay between LPs is used to determine whether an event is safe to execute. The latter is similar to the rule in CMB protocol; however in the implementation BL protocol uses a distance matrix instead of using null messages. To maintain its ordering, BL protocol uses barrier synchronization because the global clock (for imposing bounded lag restriction) and the minimum propagation delay must be broadcasted to all LPs. The algorithm is given in Figure 4.

```
BL PROTOCOL
1.  initialization
2.  run all LPs

LOGICAL PROCESS
3.  while (~stop) {
4.     β ← min {∀lp∈LP, e = head(lp.EL) •
           e.ts + d(lp, this)}
       γ ← min{∀lp∈LP, e = head(EL) •
           e.ts+d(this,lp) + d(lp,this)}
       α ← min {β, γ}
5.     barrier synchronization
6.     E ← {∀e∈EL • e.ts ≤ min(α,
           global_clock + W)}
7.     EL ← EL - E
8.     while (E ≠ ∅) {
9.        e ← head(E)
10.       E ← E - {e}
11.       {IE, EE} ← execute (e)
12.       local_clock ← e.ts
13.       EL ← EL ∪ IE
14.       Send(EE)
15.    }
16.    stop ← g()
17.    barrier synchronization
18.    global_clock ← min {∀lp∈LP •
         lp.local_clock}
19.    barrier synchronization
20. }
```

Figure 4. Algorithm of Bounded-lag Protocol

There are two main processes: the nomination of safe events (lines 4-7) and the execution of safe events (lines 8-15). The lookahead between any two LPs is stored in a distance matrix *d*. Based on the distance matrix, an LP (denoted by *this* in Figure 4) determines α, i.e. the earliest time when its system state can be affected by other LP (line 4). The barrier synchronization (line 5) ensures that all LPs receive α before continuing to the next line. Each LP identifies its safe events based on this rule: events with a timestamp less than α and within a time window of *W* are safe to process (line 6). *W* is termed as BL size in [14]. Line 7 removes all safe events from EL for execution. Next, safe events are executed in lines 8-15.

The barrier synchronization in line 17 is used to ensure that all LPs have processed their safe events before the time window is moved. Line 18 computes the global clock as the minimum of all LPs' local clock. This process is repeated until the stopping condition is met.

Lemma 3. *BL protocol implements an event order whereby event x is ordered before event y if:*

1. *$y.pred = x$, or*
2. *$x.ts + la < y.ts$, or*
3. *$\lfloor x.ts/W \rfloor < \lfloor y.ts/W \rfloor$.*

Proof. In line 7, α returns the smallest timestamp of an unprocessed event *x* (plus lookahead) that may be sent to a particular LP (Figure 4 line 4). Line 9 shows that if event *y* in LP_i can be executed in parallel with event *x* from another LP, then $y.ts \leq \alpha$ (i.e. $x.ts + la$) and both *x* and *y* must be in the same time window of size *W*. Therefore, event *x* is executed before event *y* only if $x.ts + la < y.ts$ or events *x* and *y* are in two different time windows of size *W* is true (of course the time window of *x* should be earlier than the time window of *y*). ❑

2.3.4 Time Warp Protocol

Jefferson proposed Time Warp (TW) protocol imposes that if event *x* causes event *y*, then the execution of event *x* must be completed before the execution of event *y* starts [12]. The definition of "*x* causes y" follows the relation *happened before* in [13]. To implement this ordering, TW protocol uses what is called local control mechanism (rollback and state saving) and global control mechanism (global clock calculation and fossil collection).

```
TIME WARP PROTOCOL
1. initialization
2. run all LPs

LOGICAL PROCESS
3. while (~stop) {
4.   do {
5.     m ← head(IB)
6.     if (m.ts < local_clock) {
7.       if (((m ≠ anti_message) and
         dual(m) ∉ IQ) or
         ((m = anti_message) and
         dual(m) ∈ IQ)) RollBack()
8.     }
9.     if (dual(m) ∈ IQ) Annihilate(m)
       else IQ ← IQ ∪ {m}
10.    IB ← IB - {m}
11.  } while ((m = anti_message) and
     (IB ≠ ∅))
12.  if (m = anti_message) e ← head(EL)
13.  else {
14.    if (m.ts < head(EL).ts) e ← m
15.    else {e ← head(FEL);
       EL ← EL - (e}; EL ← EL ∪ {m}}
16.  }
17.  {IE, EE} ← execute (e)
18.  local_clock ← e.ts
19.  EL ← EL ∪ IE
20.  StateSaving()
21.  Update(global_clock)
22.  FossilCollection()
23.  Send(EE)
24.  stop ← g()
25.}
```

Figure 5. Algorithm of Time Warp Protocol

Each LP stores all incoming events in an input buffer (IB) which is sorted based on the timestamp of the incoming events. Lines 4-11 find the first real event *m*. Line 5 retrieves an event *m* with the smallest timestamp for execution. Line 6 checks if *lcc* is violated. Line 7 detects whether rollback has to be done. If *m* is an anti-message, line 9 will annihilate the associated event that has to be cancelled; otherwise, it will add *m* to a list called input queue (IQ). IQ is used to store the history of all incoming messages (processed and unprocessed). Line 10 removes *m* from IB. Lines 12-15 retrieve an event *e* which has the smallest timestamp from the EL and choose the event with a smaller timestamp, between *m* and *e*. Line 17 executes the chosen event. This execution may produce a set of internal events (IE) and a set of external events (EE). Line 18 updates the local clock and line 19 updates the event list (EL). Line 20 saves the state of an LP. The global clock is updated in line 21. Events with timestamp less than the global clock will never be rollbacked. These events are called *committed events*. Hence, memory allocated to committed events can be reclaimed with the fossil collection process in line 22. Line 23 sends out the external events. Lastly, line 24 checks the stopping condition.

Lemma 4. *Time Warp protocol implements a partial event order.*

Proof. The rollback process ensures that all events in the same LP are executed in timestamp order. This implies that event *x* is ordered before event *y* if $y.pred = x$. The insertion of internal events to EL and transmission of external events are done after the event execution in line 17. This ensures that event *x* is ordered before event *y*, if $y.ante = x$. ❑

2.3.5 Bounded Time Warp Protocol

The Bounded Time Warp (BTW) protocol [21] is proposed to limit the degree of optimism in TW protocol by setting a bound on how far an LP can advance ahead of other LPs. This is accomplished by setting a time window (*W*). LPs are allowed to optimistically process events ahead of the global clock (*GVT*) but bounded by the time window *GVT+W*. No LP can advance beyond *GVT+W* before all LPs have reached this boundary.

Lemma 5. ***BTW protocol** imposes that event x is ordered before event y in if:*

1. *$y.pred = x$, or*
2. *$y.ante = x$, or*
3. *$\lfloor x.ts/W \rfloor < \lfloor y.ts/W \rfloor$*

Proof. Without time window, BTW protocol is the same as TW protocol hence the ordering rules of partial event order hold, i.e. event x is ordered before event y if $y.pred = x$ or $y.ante = x$. The additional time window synchronization imposes that the partial event order is applied to a set of events that occur within the time window. Therefore, if event x occurs within a time window that is earlier than the time window of event y, event x will be executed before event y. ❑

We summarize the formalization of the event orderings discussed in Figure 6. The ordering rules of each event order are shown in the form of x is ordered before y (denoted by $x \Rightarrow y$) if a list of conditions hold. A simulator implements a certain event order. The arrow from event ordering R in simulation model to simulator S denotes that S implements R.

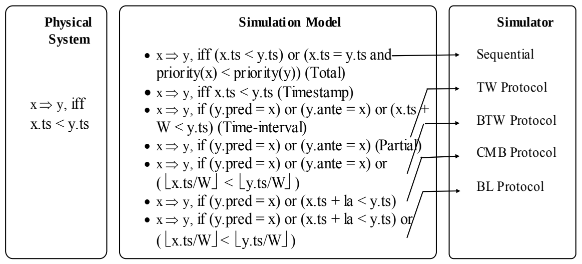

Physical System
x ⇒ y, iff x.ts < y.ts
Simulation Model
x ⇒ y, iff (x.ts < y.ts) or (x.ts = y.ts and priority(x) < priority(y)) (Total)
x ⇒ y, iff x.ts < y.ts (Timestamp)
x ⇒ y, if (y.pred = x) or (y.ante = x) or (x.ts + W < y.ts) (Time-interval)
x ⇒ y, if (y.pred = x) or (y.ante = x) (Partial)
x ⇒ y, if (y.pred = x) or (y.ante = x) or (⌊x.ts/W⌋ < ⌊y.ts/W⌋)
x ⇒ y, if (y.pred = x) or (x.ts + la < y.ts)
x ⇒ y, if (y.pred = x) or (x.ts + la < y.ts) or (⌊x.ts/W⌋ < ⌊y.ts/W⌋)
Simulator
Sequential
TW Protocol
BTW Protocol
CMB Protocol
BL Protocol

Figure 6. Summary on Event Ordering Formalization

3. Strictness of Event Ordering

To compare the degree of event dependencies among different event orders, we propose a relation *stricter*. The term stricter is borrowed from memory consistency model [6]. In memory consistency model, stricter relation is used to compare different models by considering the set of possible outcomes that is allowed by each model for a given set of instructions. In simulation event ordering, we consider the set of events that have to be executed one after another due to the ordering rules imposed by an event order for a given set of events.

Definition 11. *An event order R_1 is* ***stricter*** *than event order R_2 if for any set of events E, $R_2 \subseteq R_1$. An event order R_1 is* ***incomparable*** *to event order R_2 if we can find two sets of events E_1 and E_2, such $R_2 \subseteq R_1$ is true for E_1 but $R_2 \subseteq R_1$ is not true for E_2.*

Two events are concurrent in an event order if the event order does not impose any ordering on them. Definition 11 implies that a stricter event order produces less number of concurrent events than a less strict event order (or at most the same number of concurrent events). Since concurrent events can be executed in parallel, a stricter event order produces less event parallelism. To quantify the degree of event dependency, we propose the measure of *strictness*. Since relation stricter is built based on set inclusion, the strictness of event order R is quantified based on the number of elements in S_R as shown in Definition 12.

Definition 12. *The strictness of an event order R (ς_R) is defined as $\| S_R \| / \| S_{tot} \|$ where $\| S_R \|$ and $\| S_{tot} \|$ is the size of the set of comparable (or non-concurrent) events ordered by R and the total event order respectively.*

Since total event order is the strictest event order, we normalize the number of elements in S_R with the number of comparable elements in total event order (S_{tot}). Hence, the strictness of an event order ranges from zero when $S_R = \varnothing$ and one when R is the total event order. To measure $\| S_R \|$ for a given set of events E, we have to determine for any two events x and $y \in E$ whether $(x, y) \in S_R$ based on the ordering rules of the event order. This process is computationally expensive especially for a large number of events. Since $(x, y) \in S_R$ implies that event y cannot be executed before the execution of event x completes, in our experiments we measure the number of events that are ready for execution but cannot be executed because of the ordering rules imposed by the event order.

3.1 Strictness Analysis

Event order R_2 is stricter than event order R_1 implies that for any two distinct events x and y, if x is ordered before y in R_1 then x is also ordered before y in R_2, but not vice-versa. Therefore, to prove whether an event order is stricter than another event order, we show that the ordering rule of one event order is a subset of the other event order. If the ordering rule of event order R_1 is a subset of event order R_2, then definitely if x is ordered before y in R_1 then x is also ordered before y in R_2. Using this approach, in the following theorems, we establish the relationship of the simulation event orderings summarized in Figure 7.

Theorem 1.

a) *Total event order is stricter than TS event order.*
b) *The event order of BL protocol is stricter than the event order of CMB protocol*
c) *The event order of BTW protocol is stricter than partial event order.*

Proof. The proofs are derived by comparing their properties in Figure 6. If the property of an event order R_1 is a subset of the property of event order R_2, then R_2 is stricter than R_1. ❑

Theorem 2. *TS event order is stricter than BL event order.*

Proof. In TS event order, $x \Rightarrow y$, iff $x.ts < y.ts$. On the other hand, in BL protocol, $x \Rightarrow y$, if ($y.pred = x$) or ($x.ts + la < y.ts$) or ($\lfloor x.ts/W \rfloor < \lfloor y.ts/W \rfloor$). These rules can only be true if $x.ts < y.ts$. Therefore, if $x \Rightarrow y$ in BL protocol, then $x \Rightarrow y$ is true in TS event order, but not the converse. Hence, timestamp event order is stricter than the event order of BL protocol. ❑

Lemma 6. $\forall x, y \in E$, $\{y.ante = x\} \subseteq \{x.ts + la \le y.ts$ and $x.lp \in SENDER(y.lp)\}$.

Proof. From the definition of *SENDER* list and lookahead, if *y.ante* = *x* then *x.lp* must be in the *SENDER* list (i.e. $x.lp \in SENDER(y.lp)$) and the timestamp difference between *x* and *y* must be greater than the lookahead *la* (i.e. $x.ts + la < y.ts$). However, it is possible that $x.lp \in SENDER(y.lp)$ and $x.ts + la < y.ts$ is true but $y.ante \neq x$. ❑

Theorem 3. *The event order of CMB protocol is stricter than partial event order.*

Proof. Both have two ordering rules (Figure 8). The first rule is the same, i.e. $x \Rightarrow y$ if $y.pred = x$. In the second rule, partial event order imposes $x \Rightarrow y$ if $y.ante = x$ whereas CMB protocol imposes that $x \Rightarrow y$ if $x.ts + la < y.ts$. Lemma 6 shows that the second rule of partial event order is a subset of the second rule of CMB protocol, therefore, the event order of CMB protocol is stricter than partial event order. ❑

Theorem 4. *The event order of BL protocol is stricter than the event order of BTW protocol.*

Proof. Both have three ordering rules (Figure 8) and two of them are the same, i.e. $x \Rightarrow y$ if $y.pred = x$ or $\lfloor x.ts/W \rfloor < \lfloor y.ts/W \rfloor$. The other rule is different, BTW protocol imposes $x \Rightarrow y$ if $y.ante = x$ whereas BL protocol imposes that $x \Rightarrow y$ if $x.ts + la < y.ts$. Based on Lemma 6, BL protocol imposes a stricter event order than BTW protocol for the same window size *W*. ❑

Figure 7 shows the spectrum of event orders based on our proposed stricter relation. BL, BTW and CMB refers to the event ordering of BL protocol, BTW protocol and CMB protocol respectively. An arrow from event order R_1 to event order R_2 denotes that R_1 is stricter than R_2. Stricter relation is transitive and the arrows can be traversed transitively as well. Sequential simulation implements total event order and the remaining event orders belong mainly to parallel and distributed simulation.

Depending on its window size, the relative position of TI event order can be anywhere between timestamp event order and partial event order. If TI event ordering uses a window size of zero, then it becomes a timestamp event ordering. Similarly, there is a constant *c* such that $0 < c < W$ where time-interval event ordering becomes partial event ordering (*W* is the window size) as shown in Theorem 5. This property is useful in strictness analysis because we can create different points (representing different event orderings) between timestamp event ordering and partial event ordering by changing the value of *W*.

Theorem 5. *For a given set of events E, there is a constant c such that* $0 < c < W$*, where a TI event order will become a partial event order.*

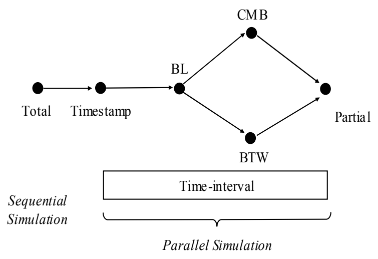


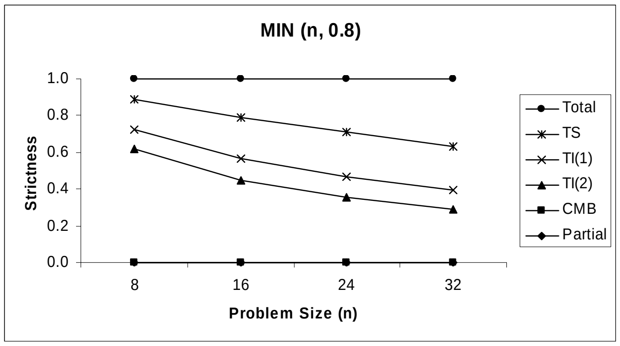
Figure 7. Spectrum of Event Orders and its Strictness

Proof. To prove this, we show that if $0 < c < W$, the third rule of time-interval event order (i.e. $x.ts + W < y.ts$) is redundant. Let *a* and *b* be two distinct events in *E* where $b.pred \neq a$ and $b.ante \neq a$ and $b.ts - a.ts = c$ is the largest. If $W > c$, then the rule $x.ts + W < y.ts$ will produce an empty set. Hence, only the first two rules (*y.pred* = *x and y.ante* = *x*) determines the ordering, resulting in TI event order with $W > c$ and partial event order producing exactly the same event ordering. ❑

4. Empirical Result

We measure the strictness of five event orders (i.e. total, timestamp, time-interval, CMB and partial) using two benchmarks:

a) Multistage Interconnected Network (MIN) represents an *open system* [19]. It is parameterized by the number of service centers (*n*), and traffic intensity (ρ).
b) Parallel Hold (PHOLD) represents a *closed system* with multiple *feedbacks* [8]. It is parameterized by the number of service centers (*n*) and job density (*m*).



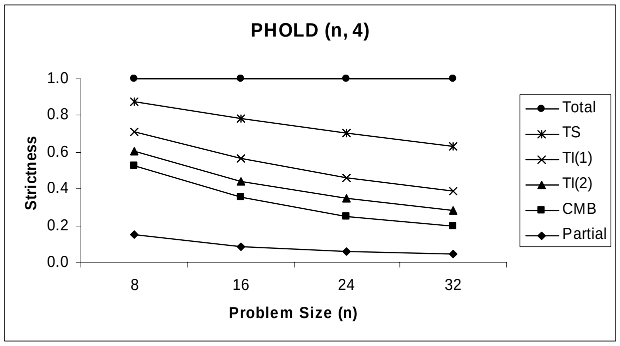


Figure 8. Strictness of Event Orderings

We measure the strictness of the event orderings using TSA [17]. The simulation duration is set at 100,000 timestamp units. Figure 8 shows the strictness of event orderings as problem size increases. First, the result shows that the strictness value is between 0 and 1, where total event order is the strictest event order. Second, the figure reveals that partial event order, CMB event order, timestamp event order and total event order are in increasing order of strictness. This confirms their positions on the event orders spectrum in Figure 7. The time-interval event order with window size of 1 and 2 represent two event orderings with different degrees of strictness. As we reduce the window size, the curve for time-interval event order moves towards timestamp event order and conversely, when we increase the window size, it moves towards the partial event order.

As problem size increases and consequently the number of events, strictness reduces. This is due to the higher probability of concurrent (non-comparable) events in the benchmarks. The strictness measure shown also reflects that the degree of event dependency in closed system is higher than in open system. Misra reported that CMB protocol can achieve optimum performance for a tandem topology and any acyclic topology [15]. Our result confirms this, i.e., strictness of the CMB (and partial) protocols for the open MIN(n, 0.8) system is lower than in the closed PHOLD(n, 4) system with multiple feedbacks.

5. Conclusions

The main contribution of this paper is the formalization of simulation event ordering based on partially ordered set theory. First, we characterized simulation performance along the three natural boundaries in simulation modeling and analysis namely: *physical system*, *simulation model* and *simulator*; and formalized the event orderings in each of the layers. Events in a physical system are ordered based on their time of occurrences. In simulation, different event orderings can be used to simulate the physical system. In the implementation, the simulator ensures that the chosen event ordering is maintained throughout a simulation run. We extract and formalize the event orderings of both sequential and parallel simulation. To compare the event dependency among different event ordering, we propose the stricter relation and to quantify the degree of event dependency a new strictness measure is proposed.

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