UIT2206 The Importance of Being Formal

Midterm, 19/3/2014

This test has nine questions on eight pages. Please answer all questions in the spaces provided after each (sub)question.

Enter your matriculation number here:

Traditional Logic

Question 1:   (6 marks)

(a) Consider the following categorical proposition:
No Mermaids are Mermaids.
Give a model in which this proposition holds or a reason why there is no such a model.

(b) Consider the following categorical proposition:
Some Werewolves are not Werewolves.
Give a model in which this proposition holds or a reason why there is no such a model.

Question 2:   (6 marks) Consider the following six categorical propositions:

- Some Men are Rich.

- Some Men are not Rich.

- Some Rich are Happy.

- Some Rich are not Happy.

- Some Men are Happy.

- Some Men are not Happy.

Give a model in which all six propositions hold.

Question 3: (8 marks) Logicians have discussed what shall be the most intuitive semantics for universal affirmative propositions. The lectures followed one of the proposed interpretations. An alternative interpretation considers a proposition of the form `All A are B` to be false in models $\mathcal{M}$ in which $A^{\mathcal{M}} = \emptyset$. In all other models, the semantics of universal affirmative propositions is as described in the lectures. This alternative interpretation is said to give *existential import* to the subject of the proposition.

Indicate which of the following syllogisms change their validity if existential import is given to the subject of universal affirmative propositions. You do not need to give reasons for your judgment.

(a)   All humans are mortal.
                All Greeks are humans.
                Therefore, Some Greeks are mortal.

(b)   All rabbits are furry.
                Some pets are rabbits.
                Therefore, Some pets are furry.

(c)   No reptiles are furry.
                All snakes are reptiles.
                Therefore, No snakes are furry.

(d)   No flowers are animals.
                All flowers are plants.
                Therefore, Some plants are not animals.

Propositional Logic

Question 4: (5 marks) Indicate which of the following formulas are valid. You do not need to give reasons for your judgment.

(a) $\neg\top \lor \top$

(b) $\neg\bot \land \bot$

(c) $\neg\top \land \top$

(d) $\neg\top \to \bot$

(e) $\top \to \neg\top$

Question 5: (4 marks)
Is every invalid formula unsatisfiable? Give a short explanation of your answer, based on the definition of the adjectives involved.

Question 6: (5 marks) Assume the following entailment holds:

$$\phi \models \psi$$

Which one of the following statements is correct? You do not need to give reasons for your judgment.

(a) If ψ is unsatisfiable, then ϕ is unsatisfiable.

(b) If ψ is valid, then ϕ is valid.

(c) If ϕ is unsatisfiable, then ψ is unsatisfiable.

(d) If ϕ is valid, then ψ is valid.

(e) If ϕ is invalid, then ψ is unsatisfiable.

Predicate Logic

Question 7: (8 marks) Let P be a predicate symbol of arity 0, Q be a predicate symbol of arity 2 and f be a function symbol of arity 2. Consider the formula

$$\phi = \exists w((P \vee (\forall v Q(f(x, v), w))) \wedge (\forall z Q(f(w, x), z)))$$

(a) Draw the parse tree of ϕ and indicate the free and bound variables in this parse tree.

(b) Give the result of the substitution $[x \Rightarrow f(x, x)]\phi$.

Question 8:   (4 marks) Consider the following model $\mathcal{M}$ for predicate logic with the binary relation R.

- Let $U^{\mathcal{M}} = \{a, b, c, d\}$

- Let $R^{\mathcal{M}} = \{(a, b), (b, c), (c, d), (d, a)\}$

For the following formulas ϕ, indicate (without explanation),
whether $\mathcal{M} \models \phi$ or $\mathcal{M} \not\models \phi$.

(a) $\exists x (R(x, x)))$

(b) $\forall x (\neg R(x, x))$

(c) $\exists x (\forall y (R(x, y)))$

(d) $\forall x (\exists y (R(x, y)))$

Question 9:   (4 marks) In predicate logic, the universe of a model cannot be empty. If we remove this restriction, the validity of some formulas changes. Give an example for such a formula and a short explanation.