CTL Model Checking

Abhik Roychoudhury
CS 5219
National University of Singapore

News
- Turing Award 2007 (announced Feb 08)
  - Clarke, Emerson, Sifakis
    - For their role in developing Model-Checking into a highly effective verification technology, widely adopted in the hardware and software industries.

http://awards.acm.org/homepage.cfm?art=at6&awd=148

Award in 1996 for temporal logics to Amir Pnueli

Model Checking
- Specification (e.g. Promela)
- Or Software
- Temporal Logic property (e.g. in CTL or LTL)
- Finite State Transition Sys
- MC
- YES
- No, counterexample evidence (e.g. trace for LTL)

CTL Model Checking
- Inputs:
  - Finite state Kripke Structure $M = (S, I, \rightarrow, L)$
    - $S$ (set of states),
    - $I \subseteq S$ (set of initial states),
    - $\rightarrow$ (transition relation),
    - $L$ (function labeling atomic propositions to states)
  - CTL formula $\phi$ (with atomic propositions corresponding to those appearing in $M$)
- Output:
  - Whether for all $s$ in $I$, we have $M, s \models \phi$

Example: A microwave oven
- Assignment of start, heat are shown
- AG (start ⇒ AF heat)

Example
- AG (start ⇒ AF heat)
  - For any reachable state, if start holds, then along all outgoing paths, heat eventually holds.
- Violated if:
  - $\exists$ a reachable state $s$ where start holds
  - $\exists$ an acyclic path from $s$ to $s''$ in which heat does not hold in any state
  - And there is a cycle containing $s'$ such that heat does not hold in all states of the cycle.
- Can we find such a cycle in the given model?
**Example: A microwave oven**

![Diagram of microwave oven]

Assignment of start, heat are shown

**CTL Model Checking**

- A systematic way of doing the reasoning performed in our example for all CTL formulae.
- Recall set of all CTL formulae:
  - $\varphi = \text{Prop} \mid \neg \varphi \mid \varphi \land \varphi \mid \AX \varphi \mid \EX \varphi \mid \AF \varphi \mid \EF \varphi$
  - $\mid \EG \varphi \mid \AG \varphi \mid \E(\varphi \land \varphi) \mid \A(\varphi \land \varphi)\mid \E(\varphi R \varphi) \mid \A(\varphi R \varphi)\mid \E(\varphi R \varphi) \mid \A(\varphi R \varphi)$
- Among the temporal operators, only consider:
  - EX, EG, EU (along with $\neg$ and $\land$)

**CTL temporal operators**

- $\AX \varphi = \neg \neg \AX \varphi = \neg \EX \neg \varphi$
- $\AG \varphi = \neg \neg \AG \varphi = \neg \EF \neg \varphi$
- $\EF \varphi = \E(\text{true} \lor \varphi)$
- $\AF \varphi = \neg \EG \neg \varphi$
- $\A(\varphi R \Psi) = \neg \neg \A(\varphi R \Psi) = \neg \E(\neg \varphi \lor \neg \Psi)$
- $\A(\varphi U \Psi) = \neg \E(\neg \varphi R \neg \Psi)$
- What about $\E(\varphi R \Psi)$ ??

**Model Checking Algorithm**

- Pre-processing: Rewrite the CTL formula to be verified to contain EX, EG, EU, $\land$, $\neg$
- For all sub-formulae $x$ of the re-written formula $\varphi$, find the set of states satisfying $x$ in the given model $M$.
- The above step finally computes the set of states in $M$ which satisfy $\varphi$, call it $S_{\varphi}$.
- Check whether all initial states of $M$ are contained in $S_{\varphi}$.

**Word of caution**

- The pre-processing of the formula is only being done to simplify the presentation of the MC algorithm.
- Reduce number of cases to consider.
- We can develop customized algorithms for each of the 10 CTL operators, and then apply them in a bottom-up recursive fashion as we will be doing now.

**Example**

- $M = \text{the model of microwave oven given earlier}$
- $\varphi = \AF \text{(start } \Rightarrow \text{AF heat)}$
- $\neg \EF \neg(\neg \text{start } \lor \AF \text{ heat)}$
- $\neg \EF (\text{start } \land \neg \AF \text{ heat)}$
- $\neg \EF (\text{start } \land \EG \neg \text{ heat)}$
- $\neg \E(\text{true } \lor (\text{start } \land \EG \neg \text{ heat}))$
- Now, how to compute the set of states in $M$ satisfying this transformed formula?
Example

Essentially by bottom-up traversal of the formula

Example

Steps in the traversal - Base case

Example

Steps in the formula traversal

Example

Steps in the formula traversal

Example

Steps in the formula traversal

Example

Steps in the formula traversal

Example

Steps in the formula traversal
**Example**

Steps in the formula traversal

- $S_5 = S - S_4$
- $S_4 = ???$
- $S_1 = S$
- $S_2 \land S_4$
- $S_2 = ???$
- $S_3 = S - S_3$

Check whether $S_6$ contains all initial states of $M$

**Bottom-up formula traversal**

Base case: Atomic propositions

- $p$

Boolean operators: $\land$, $\neg$

- $S - S_1$
- $S_1 \land S_2$
- $S_1$

**Questions Remaining**

Temporal operators: $\text{EX}$, $\text{EU}$, $\text{EG}$

- $\text{EX}$
  - $s \triangleright s_1$, $s_1 \in S_1$

- $\text{EU}$
  - $??$

- $\text{EG}$
  - $??$

**EU**

- Inputs:
  - Kripke Structure $M = (S, I, \rightarrow, L)$.
  - CTL formulae $\phi$ and $\Psi$.
  - $S_\phi$, set of states satisfying $\phi$ in $M$.
  - $S_\Psi$, set of states satisfying $\Psi$ in $M$.

- Output:
  - Set of states satisfying $E(\phi U \Psi)$ in $M$.

- Technique:
  - *Traversing the states (and transitions) of $M$.*

**E(\phi U \Psi): Intuition**

- Result := $S_\Psi$;
- Temp := $S_\Psi$;
- while Temp $\neq$ empty do
  - pick $s$ $\in$ Temp; Temp := Temp $-$ {$s$};
  - Backstep := {$s_1$ | $s_1 \rightarrow s$, and $s_1 \in S_\phi$};
  - Temp := Temp $\cup$ Backstep;
  - Result := Result $\cup$ Backstep;
- endwhile;
- return Result;

**E(\phi U \Psi): Algorithm**

- Result := $S_\Psi$;
- Temp := $S_\Psi$;
- while Temp $\neq$ empty do
  - pick $s$ $\in$ Temp; Temp := Temp $-$ {$s$};
  - Backstep := {$s_1$ | $s_1 \rightarrow s$, and $s_1 \in S_\phi$};
  - Temp := Temp $\cup$ Backstep;
  - Result := Result $\cup$ Backstep;
- endwhile;
- return Result;
**EG**

- **Inputs:**
  - Kripke Structure $M = (S, I, \rightarrow, L)$.
  - CTL formulae $\phi$.
  - $St_\phi$, set of states satisfying $\phi$ in $M$.
- **Output:**
  - Set of states satisfying $EG \phi$ in $M$.
- **Technique:**
  - *Traversing the states (and transitions) of $M$.*

**EG $\phi$: Intuition**

![Diagram of Kripke Structure](image)

**EG $\phi$: Algorithm**

- **Result** := $St_\phi$;
- **repeat**
  - $Temp := \{ s | s \in Result$, and $\forall s1. s \rightarrow s1 \Rightarrow s1 \notin Result \};$
  - $Result := Result - Temp;$
- **until** $Temp = empty;$
- **return** Result;

**How to make it more efficient**

- We initialize $St_{EG\phi} = St_\phi$.
- For each state in $St_\phi$, we check the out-edges. Many of the destination states are not in $St_\phi$, so cannot satisfy $EG\phi$.
- It suffices to consider a reduced Kripke Structure $M'$ constructed from $M$ such that
  - All states of $M$ which satisfy $\phi$ are retained.
  - All other states and transitions are deleted.
- For any $s$, we have $M,s \models EG\phi$ if and only if
  - $s$ is a state in $M'$
  - $s$ reaches a state $s'$ in $M'$ where $s'$ loops back to itself.

**Efficient computation**

- **Input:** $M = (S, I, \rightarrow, L)$, $St_\phi$
- **Output:** $St_{EG\phi}$
- **Technique:**
  - Compute $M'=(S', I', \rightarrow', L')$ from $M$ by keeping only nodes in $St_\phi$.
  - Take all nodes in nontrivial SCCs of $M'$.
  - While $Temp = empty$ do
    - Pick $s \in Temp$;
    - $Temp := Temp \setminus \{s\}$;
    - $St_{EG\phi} := St_{EG\phi} \cup \{ t | t \rightarrow s \land t \notin St_{EG\phi} \}$;
    - $Temp := Temp \cup \{ t | t \rightarrow s \land t \notin St_{EG\phi} \}$;
  - end

**Summary, Exercises**

- We have only presented model checking as a decision procedure.
- Other issues such as counter-example computation not shown.
- Direct iterative algorithms given only for EU, EG
- What about EF, AF, AG etc.?
- Algorithmic complexity of the iterative algorithms discussed in today’s lecture.