Revision of Timing Analysis

CS4271
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Q1. Timing Schema

Consider the following program fragment that computes in z the product of x and y. Thus, x and y serve as inputs to the program fragment, and z serves as the output of the program fragment. Both the inputs are positive integers, given as unsigned 8 bit numbers (when represented in binary). Using Timing Schema WCET analysis method discussed in class, derive the maximum execution time of the program fragment. Each assignment/return/condition-evaluation takes 1 time unit.

\[ z = 0; \\
\text{while} (x > 0) \{
\text{if} (x \% 2 != 0) \{ 
 z = z + y; \\
\}
 y = 2 * y; x = x/2; \\
\}
\text{return} z; \]

Q2: ILP Modeling

Formulate the maximum execution time estimation of the program fragment in Question 3(A) using Integer Linear Programming (ILP). Clearly show the objective function and all constraints. Your ILP problem should only perform program path analysis and not micro-architectural modeling. The estimate produced by your ILP problem should be as tight as possible.

Answer to Q1

\[ T(\text{Program}) = T(\text{x=0}) + T(\text{while}) + T(\text{return}) + 1 + T(\text{while}) + 1 = 2 + T(\text{while}) \]

To estimate the time for the while-loop, we need the loop bound LB, which here is \( \log x \). Since x is an 8 bit number, this gives us a loop bound of \( \text{LB} = 8 \).

\[ T(\text{while}) = (\text{LB}+1)(T(\text{x=0}) + \text{LB}^3(\text{T(if) + T(\text{y = x%2}) + T(\text{x=x/2})}) = 9*1 + 8^3(1+1+1+1) \]

Now, \( T(\text{if}) = T(\text{x%2 = 0}) + T(\text{y = 2*y}) = 1 \) + 1 = 2

So, \( T(\text{while}) = 9*1 + 8^3(2 + 1 + 1) = 9 + 32 = 41 \text{ time units} \)

So, \( T(\text{program}) = 2 + T(\text{while}) = 2 + 41 = 43 \text{ time units} \).

Answer to Q2

Based on the control flow graph, the flow constraints are as follows. \( N_1 \) is the execution count of basic block 1, and \( E_{ij} \) is the execution count of the edge from basic block i to basic block j.

\[ 1 = N_2 = E_{21}, \quad E_{12} + E_{13} + E_{14} + E_{15} = N_1, \quad N_6 = E_{16} + E_{36}, \quad N_5 = E_{25} + E_{35}, \quad N_6 = E_{26} + E_{36}, \quad E_{15} + E_{16} + E_{17} = 1 \]

The loop bound accounts for the additional constraint \( E_{12} \leq 4 \).

Answer to Q2

The objective function is \( c_1*N_1 + c_2*N_2 + c_3*N_3 + c_4*N_4 + c_5*N_5 + c_6*N_6 \)

\( c_1 \) is the execution cost of basic block 1, \( c_2 \) is the execution cost of basic block 2 and so on.

Since assignments/conditions/returns all take 1 time unit, we get

\[ c_1 = 1, \quad c_2 = 1, \quad c_3 = 1, \quad c_4 = 1, \quad c_5 = 2, \quad c_6 = 1 \]

The objective function can now be maximized w.r.t. flow constraints and loop bounds. We will use an ILP solver for this purpose.

The result from the ILP problem given above should be same as the result returned from timing schema.
Q3: Comparison

- Also, comment on how the estimate from your ILP problem will compare with the estimate you produced using Timing schema.

- Answer: The result from the ILP problem is exactly the same as the result returned from timing schema.