(1) Are the two following Linear time Temporal Logic formula equivalent? If yes, give a proof. If not, construct example traces to show that they are not equivalent.

\[ F(p \mathbf{U} q) \Leftrightarrow Fp \mathbf{U} Fq \]

You can assume that \( p \) and \( q \) are atomic propositions.

(2) In class, we discussed the nested depth-first search algorithm implemented inside the model checker SPIN. Among other things, this allows us to easily retrieve the counter-example trace from the stack. Suppose we implemented breadth-first search with queues instead for the purpose of model checking. Will the task of counter-example computation become any more difficult? Explain your answer.

(3) Recall the definition of the Until operator \( \mathbf{U} \) in Linear-time temporal logic (LTL). Let us now define a new until operator \( \mathbf{U}_1 \) as follows:

\[ M, \pi \models \varphi \mathbf{U}_1 \psi \Leftrightarrow \text{if there exists a } k \geq 0 \text{ such that } M, \pi^k \models \psi \text{ then for all } 0 \leq j < k \text{ we have } M, \pi^j \models \varphi \]

The notation \( \pi^k \) was discussed in class (and also appears in the textbook). Express \( \varphi \mathbf{U}_1 \psi \) as a Linear-time temporal logic (LTL) formula and give explanation for your answer. You may assume that \( \varphi, \psi \) are arbitrary LTL properties.

(4) Assume \( p \) is an atomic proposition. Describe the following property in LTL: “along any path, a state satisfying \( p \) occurs at most once”. Explain your answer.