Quiz 11: Make Up Quiz. For true/false questions, use (A) True and (B) False. For a set $A$, $|A|$ denotes the number of elements of $A$. $\mathbf{R}$ is the set of real numbers, $\mathbf{Q}$ is the set of rational numbers, $\mathbf{Z}$ is the set of integers, $\mathbf{Z}^+$ is the set of positive integers. If you are making up for Quiz 1, you should do only the first 10 questions.

1. State the truth value of the statement “if $1 > 2$ then $2 > 1$.”

2. Let $P$, $Q$, $R$ be statement forms and it is known that “$P, Q, : \vdash R$” is a valid argument form. What can be said about the statement form “$P \land Q \rightarrow R$”?
   (A) A true statement. (B) A false statement. (C) A tautology. (D) A contradiction.

3. The statement “$\forall x \in \mathbf{Z}, P(x) \rightarrow Q(x)$” is true. The truth set of of $P(x)$ is $\mathbf{Z}^+$. What is the truth value of “$P(0) \rightarrow Q(0)$”?

4. Let $A = \{0, 1, 2, 3, 4, 5\}$. How many minimal elements are there in the poset $(P(A) \setminus \{\emptyset, A\}, \subseteq)$?
   (A) 0. (B) 1. (C) 5. (D) 6. (E) None of the above.

5. If $Q^1 = Q$, $Q^{n+1} = Q \times Q^n$, then $|\bigcup_{n=1}^{\infty} Q^n| = |\mathbf{Z}^+|$.

6. For any non-negative integer $n$, we have
   (A) $\sum_{k=0}^{n} \left( \begin{array}{c} n \\ k \end{array} \right) < 2^n$. (B) $\sum_{k=0}^{n} \left( \begin{array}{c} n \\ k \end{array} \right) = 2^n$. (C) $\sum_{k=0}^{n} \left( \begin{array}{c} n \\ k \end{array} \right) > 2^n$.

7. The set $\{(x, -x) \in \mathbf{Z} \times \mathbf{Z} : x < 0\} \cup \{(x, 2x) \in \mathbf{Z} \times \mathbf{Z} : x > 0\}$ define a function from $\mathbf{Z}$ to $\mathbf{Z}^+$.

8. For any set $A$ and $f : A \rightarrow P(A)$ with $f(x) = A \setminus \{x\}$ for any $x \in A$. Let $S = \{x \in A : x \notin f(x)\}$. What is $S$?
   (A) $\emptyset$. (B) The set $A$. (C) Cannot be determined unless $A$ is known. (D) None of the above.

9. For any non-empty set $A$, the only relation that is symmetric, anti-symmetric, and transitive is the identity relation.

10. Let $R$ be a relation on a set $A$. If $R \cap R^{-1} = \emptyset$ then $R$ is anti-symmetric.

11. Let $G = (V, E)$ be a graph and $V' \subseteq V$, $E' \subseteq E$. Then $G' = (V', E')$ is a sub-graph of $G$.

12. For any set $A$, we have $P(A \times A) = P(A) \times P(A)$.

13. The number of symmetric relations on a set $A$ is
   (A) $2^n$ where $n = \frac{|A||A|+1}{2}$. (B) $2^n$ where $n = |A|$. (C) $\frac{|A||A|+1}{2}$.
   (D) $\binom{|A|}{2}$. (E) None of the above.

14. For any graph $G$, if $G$ is complete then $G$ is not bipartite.

15. If set $A$ is infinite, then either $|A| = |\mathbf{R}|$ or $|A| = |\mathbf{Z}|$. 

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