1. Use the predicates

\[ A(x, y) : \text{x admires y} \]
\[ B(x, y) : \text{x attended y} \]
\[ P(x) : \text{x is a professor} \]
\[ S(x) : \text{x is a student} \]
\[ L(x) : \text{x is a lecture} \]

and the function constant

\[ m : \text{Mary} \]

to translate the following into predicate logic:

(a) Mary admires every professor.
(b) Some professor admires Mary.
(c) Mary admires herself.
(d) No student attended every lecture.
(e) No lecture was attended by every student.
(f) No lecture was attended by any student.

2. Consider the following formula, denoted by \( \Phi \):

\[ \neg (\forall x ((\exists y P(x, y, z)) \land (\forall z P(x, y, z)))) \]

(a) Draw the parse tree of \( \Phi \).
(b) Indicate the free and bound variables in that parse tree.
(c) List all variables which occur free and bound therein.
(d) Compute \( \Phi[t/x], \Phi[t/y], \text{ and } \Phi[t/z] \), where \( t \) equals the term \( g(f(g(y,y)),y) \). Is \( t \) free for \( x \) in \( \Phi \)? Is \( t \) free for \( y \) in \( \Phi \)? Is \( t \) free for \( y \) in \( \Phi \)? Is \( t \) free for \( z \) in \( \Phi \)?

3. Prove the following sequents in predicate logic, using natural deduction rules.

(a) \( \forall x (P(x) \land Q(x)) \vdash \forall x P(x) \land \forall x Q(x) \)
(b) \( \exists x P(x) \lor \exists x Q(x) \vdash \exists x (P(x) \lor Q(x)) \)
(c) \( \forall x \forall y P(x, y) \vdash \forall u \forall v P(u, v) \)
(d) \( \exists x \forall y P(x, y) \vdash \forall y \exists x P(x, y) \)
(e) \( P(a) \vdash \forall x (x = a \rightarrow P(x)) \)
(f) \( \forall x P(x) \rightarrow S \vdash \exists y (P(y) \rightarrow S) \) (\( S \) is a predicate with 0 arguments)