Tutorial 5

1. Unify the following pairs of atomic formulas, if possible.

   \[ p(a, x, f(g(y))) \text{ and } p(y, f(z), f(z)) \]
   \[ p(x, g(f(a)), f(x)) \text{ and } p(f(a), y, y) \]
   \[ p(x, g(f(a)), f(x)) \text{ and } p(f(y), z, y) \]
   \[ p(a, x, f(g(y))) \text{ and } p(z, h(z, u), f(u)) \]

2. Given the logic program

   \[
   \begin{align*}
   p(a, b) \\
   p(c, b) \\
   p(x, y) & \leftarrow p(x, z), p(z, y) \\
   p(x, y) & \leftarrow p(y, x)
   \end{align*}
   \]

   and the goal \( \leftarrow p(a, c) \), show that if any clause is omitted from the program, then there is no refutation. From this, prove that if a depth first search rule is used with any fixed order of the clauses, there is no refutation no matter what computation rule is used.

3. 
   (a) Write a Prolog predicate that takes a list of integers and splits it into two lists containing the odd-ranked, and the even-ranked elements of the original list, respectively.

   Sample call:
   \[
   ?- \text{split}([10,2,3,7,5,1], A, B). \]

   (b) Write a Prolog predicate that takes two \textit{sorted} lists of integers and merges them into a sorted list containing all the elements of the two lists.

   Sample call:
   \[
   ?- \text{merge}([3,5,10],[1,2,7], A). \]
   Answer: \( A=[1,2,3,5,7,10] \)

   (c) Using the \texttt{split} and \texttt{merge} predicates, write a Prolog predicate that sorts a list of integers using the \textit{mergesort algorithm}.

   Sample call:
   \[
   ?- \text{mergesort}([10,2,3,7,5,1], A). \]
   Answer: \( A=[1,2,3,5,7,10] \)