Tutorial 6

1. Model checking, 1:
   Apply the model checking algorithm to check properties \( \phi_1, \phi_2, \phi_3, \) and \( \phi_4 \) in the course notes on the first mutual exclusion model.

2. Model checking, 2:
   Apply the model checking algorithm to check properties \( \phi_1, \phi_2, \phi_3, \) and \( \phi_4 \) in the course notes on the second mutual exclusion model.

3. Model checking, 3:
   Apply the model checking algorithm to check the formula \( \phi = \text{AG}(p \rightarrow \text{A}[p \lor (\neg p \land \text{A}[-p \lor q])]) \) for the model described in the figure.

4. From graphs to SMV: Write an SMV program for the question in Exercise 3.

5. From SMV to graphs: Draw the state transition diagram associated to the following SMV program and check its CTL formula.

   \begin{verbatim}
   MODULE main
   VAR
     bit0 : counter_cell(1);
     bit1 : counter_cell(bit0.carry_out);
   \end{verbatim}
SPEC
    AG AF bit1.carry_out

MODULE counter_cell(carry_in)
VAR
    value : boolean;
    carry_out : boolean;
ASSIGN
    init(value) := 0;
    next(value) := value + carry_in mod 2;
    carry_out := value & carry_in;

For questions 6-7: One can define fixed point operators on a (possibly infinite) tree as follows. By $Z, Z'$, etc. we denote subsets of positions in the tree. Let $\mathcal{F}$ be a mapping $Z \mapsto \mathcal{F}(Z)$. $\mathcal{F}$ is monotone if $Z \subseteq Z'$ implies $\mathcal{F}(Z) \subseteq \mathcal{F}(Z')$; it is continuous if $\mathcal{F}(\cup_i Z_i) = \cup_i \mathcal{F}(Z_i)$, for any increasing sequence $Z_0 \subseteq Z_1 \subseteq \ldots$. For such a monotone and continuous $\mathcal{F}$ the increasing sequence

$$\emptyset \subseteq \mathcal{F}(\emptyset) \subseteq \mathcal{F}(\mathcal{F}(\emptyset)) \subseteq \ldots$$

define the least fixed point of $\mathcal{F}$

$$\mu Z. \mathcal{F}(Z) =_{def} \emptyset \cup \mathcal{F}(\emptyset) \cup \mathcal{F}(\mathcal{F}(\emptyset)) \cup \ldots$$

Similarly, for a monotone and continuous $\mathcal{F}$ the decreasing sequence ($T$ is the set of all positions in the tree)

$$T \supseteq \mathcal{F}(T) \supseteq \mathcal{F}(\mathcal{F}(T)) \supseteq \ldots$$

define the greatest fixed point of $\mathcal{F}$

$$\nu Z. \mathcal{F}(Z) =_{def} T \cap \mathcal{F}(T) \cap \mathcal{F}(\mathcal{F}(T)) \cap \ldots$$

6. **Minimal fixed point** ($\text{AF } \phi = \mu Z. \phi \lor \text{AX } Z$): Let $Y$ be the positions in a tree where $\phi$ is true and $\mathcal{G}(Z) = \{p: p$ is a position in the tree and any next position of $p$ is in $Z\}$. Show that $Z \mapsto Y \cup \mathcal{G}(Z)$ is monotone and continuous and its minimal fixed point $\mu Z. Y \cup \mathcal{G}(Z)$ represents the set of positions in the tree where $\text{AF } \phi$ holds.

7. **Maximal fixed point** ($\text{AG } \phi = \nu Z. \phi \land \text{AX } Z$): Let $Y$ and $\mathcal{G}(Z)$ be as before. Show that $Z \mapsto Y \cap \mathcal{G}(Z)$ is monotone and continuous and its maximal fixed point $\nu Z. Y \cap \mathcal{G}(Z)$ represents the set of positions in the tree where $\text{AG } \phi$ holds.