Starting the Module

Module TraditionalLogicCheat.

Defining Terms

Parameter Term : Type.

Populate the type Term with a particular instance.

Parameter cats : Term.
Parameter lions : Term.

Propositions

Record Quantity : Type :=
universal : Quantity
| particular : Quantity.

Record Quality : Type :=
affirmative : Quality
| negative : Quality.
Record CategoricalProposition : Type :=
cp {
  quantity : Quantity;
  quality : Quality;
  subject : Term;
  object : Term
}.

Example:

Definition LionsAreCats : CategoricalProposition :=
  cp universal affirmative lions cats.

Notation "'All' subject 'are' object " :=
  (cp universal affirmative subject object) (at level 50).

Notation "'No' subject 'are' object " :=
  (cp universal negative subject object) (at level 50).

Notation "'Some' subject 'are' object " :=
  (cp particular affirmative subject object) (at level 50).

Notation "'Some' subject 'are' 'not' object " :=
  (cp particular negative subject object) (at level 50).

Now you can write:

Definition LionsAreCats2 : CategoricalProposition :=
  All lions are cats.

Propositions may “hold”, which means they can be used in following proofs.

Parameter holds : CategoricalProposition -> Prop.

Example:

Axiom LionsAreCatsHolds: holds (All lions are cats).

Complement

Parameter non: Term -> Term.

Axiom (NonNon). For any term t, the term non non t is considered equal to t.

Axiom NonNon: forall t, non (non t) = t.
Conversion

Definition (ConvDef). For all terms $t_1$ and $t_2$, we define

\[
\begin{align*}
\text{convert}(\text{All } t_1 \text{ are } t_2) &= \text{All } t_2 \text{ are } t_1 \\
\text{convert}(\text{Some } t_1 \text{ are } t_2) &= \text{Some } t_2 \text{ are } t_1 \\
\text{convert}(\text{No } t_1 \text{ are } t_2) &= \text{No } t_2 \text{ are } t_1 \\
\text{convert}(\text{Some } t_1 \text{ are not } t_2) &= \text{Some } t_2 \text{ are not } t_1
\end{align*}
\]

Definition $\text{convert}: \text{CategoricalProposition} \to \text{CategoricalProposition} :=$

\[
\begin{array}{l}
\text{fun } x : \text{CategoricalProposition} \Rightarrow \\
\quad \text{match } x \text{ with} \\
\quad \quad \text{cp quantity quality subject object} \Rightarrow \\
\quad \quad \quad \text{cp quantity quality object subject} \\
\quad \text{end.}
\end{array}
\]

Axiom (ConvE1). If, for some terms $t_1$ and $t_2$, the proposition

\[
\text{convert}(\text{Some } t_1 \text{ are } t_2)
\]

holds, then the proposition

\[
\text{Some } t_1 \text{ are } t_2
\]

also holds.

Axiom (ConvE2). If, for some terms $t_1$ and $t_2$, the proposition

\[
\text{convert}(\text{No } t_1 \text{ are } t_2)
\]

holds, then the proposition

\[
\text{No } t_1 \text{ are } t_2
\]

also holds.

\[
\begin{align*}
\text{convert}(\text{Some } t_1 \text{ are } t_2) \\
\hline
\text{convert}(\text{Some } t_1 \text{ are } t_2) &\quad \text{[ConvE1]} \\
\text{Some } t_1 \text{ are } t_2 \\
\text{convert}(\text{No } t_1 \text{ are } t_2) \\
\hline
\text{convert}(\text{No } t_1 \text{ are } t_2) &\quad \text{[ConvE2]} \\
\text{No } t_1 \text{ are } t_2
\end{align*}
\]
Axiom ConvE1: 
for all subject object, 
holds (convert (Some subject are object)) 
-> 
holds (Some subject are object).

Axiom ConvE2: 
for all subject object, 
holds (convert (No subject are object)) 
-> 
holds (No subject are object).

Custom-made tactic eliminateConversion1 applies ConvE1 and then unfolds convert. Custom-made tactic eliminateConversion2 applies ConvE2 and then unfolds convert.

Contraposition

Definition (ContrDef). For all terms \( t_1 \) and \( t_2 \), we define

\[
\text{contrapose}(\text{All} \ t_1 \ \text{are} \ t_2) = \text{All non} \ t_2 \ \text{are} \ \text{non} \ t_1
\]
\[
\text{contrapose}(\text{Some} \ t_1 \ \text{are} \ t_2) = \text{Some non} \ t_2 \ \text{are} \ \text{non} \ t_1
\]
\[
\text{contrapose}(\text{No} \ t_1 \ \text{are} \ t_2) = \text{No non} \ t_2 \ \text{are} \ \text{non} \ t_1
\]
\[
\text{contrapose}(\text{Some} \ t_1 \ \text{are not} \ t_2) = \text{Some non} \ t_2 \ \text{are not} \ \text{non} \ t_1
\]

Definition contrapose: CategoricalProposition -> 
CategoricalProposition := 
fun x : CategoricalProposition => match x with 
| cp quantity quality subject object => 
  cp quantity quality (non object) (non subject)
end.

Axiom (ContrE1). If, for some terms \( t_1 \) and \( t_2 \), the proposition 
\[
\text{contrapose}(\text{All} \ t_1 \ \text{are} \ t_2)
\]
holds, then the proposition 
\[
\text{All} \ t_1 \ \text{are} \ t_2
\]
also holds.

Axiom (ContrE2). If, for some terms \( t_1 \) and \( t_2 \), the proposition 
\[
\text{contrapose}(\text{Some} \ t_1 \ \text{are not} \ t_2)
\]
holds, then the proposition 
\[
\text{Some} \ t_1 \ \text{are not} \ t_2
\]
also holds.
contrapose(All \( t_1 \) are \( t_2 \))

\[ \frac{}{\text{All } t_1 \text{ are } t_2} \] [ContrE1]

contrapose(Some \( t_1 \) are not \( t_2 \))

\[ \frac{}{\text{Some } t_1 \text{ are not } t_2} \] [ContrE2]

Axiom ContrE1:
for all subject object,
holds (contrapose (All subject are object))

\[ \frac{}{\text{holds (All subject are object).}} \]

Axiom ContrE2:
for all subject object,
holds (contrapose (Some subject are not object))

\[ \frac{}{\text{holds (Some subject are not object).}} \]

As with conversion, we have defined corresponding tactics eliminateContraposition1 and eliminateContraposition2 which allow us to rewrite the proof.

Custom-made tactic eliminateContraposition1 applies ContrE1 and then unfolds contrapose. Custom-made tactic eliminateContraposition2 applies ContrE2 and then unfolds contrapose.

Obversion

Definition (ObvDef). For all terms \( t_1 \) and \( t_2 \), we define

\[
\begin{align*}
\text{obvert}(\text{All } t_1 \text{ are } t_2) &= \text{No } t_1 \text{ are non } t_2 \\
\text{obvert}(\text{Some } t_1 \text{ are } t_2) &= \text{Some } t_1 \text{ are not non } t_2 \\
\text{obvert}(\text{No } t_1 \text{ are } t_2) &= \text{All } t_1 \text{ are non } t_2 \\
\text{obvert}(\text{Some } t_1 \text{ are not } t_2) &= \text{Some } t_1 \text{ are non } t_2
\end{align*}
\]

We first introduce a means to obtain the opposite of a quality.

Definition complement: Quality -> Quality :=

\[
\text{fun } x : \text{Quality} => \text{match } x \text{ with } \\
| \text{affirmative} => \text{negative} \\
| \text{negative} => \text{affirmative} \\
\end{\text{match}}
\]
Definition \textit{obvert}: CategoricalProposition \to CategoricalProposition := 
\text{fun} \ x : \ \text{CategoricalProposition} \ \Rightarrow \ \text{match} \ x \ \text{with} \ \\
\mid \ \text{cp} \ \text{quantity} \ \text{quality} \ \text{subject} \ \text{object} \ \\
\Rightarrow \ \text{cp} \ \text{quantity} \ (\text{complement} \ \text{quality}) \ \text{subject} \ (\text{non} \ \text{object}) \ \\
\text{end}.

\textbf{Axiom (ObvE).} If, for some proposition } p \\
\textit{obvert}(p) \\
\text{holds, then the proposition } p \text{ also holds.}

\textbf{Axiom ObvE :} \\
\text{forall catprop, holds (obvert catprop) \to holds catprop.}

\[
\begin{array}{c}
obvert(p) \\
\hline \\
[\text{ObvE}] \\
p
\end{array}
\]

Custom-made tactic \texttt{eliminateObversion} applies \texttt{ObvE} and then unfolds \texttt{obvert}.

\textbf{Syllogisms}

\textbf{Axiom (Barbara).} For all terms minor, middle, and major, if \textit{All middle are major} holds, and \textit{All minor are middle} holds, then \textit{All minor are major} also holds.

\[
\begin{array}{c}
\text{All middle are major} \quad \text{All minor are middle} \\
\hline \\
\text{All minor are major} \quad \text{[Barbara]}
\end{array}
\]

\textbf{Axiom Barbara :} \text{forall major minor middle,} \\
\text{holds (All middle are major)} \\
\text{\&/\ holds (All minor are middle)} \\
\text{\to holds (All minor are major).}

\textbf{Axiom (Celarent).} For all terms minor, middle, and major, if \textit{No middle are major} holds, and \textit{All minor are middle} holds, then \textit{No minor are major} also holds.
No middle are major  All minor are middle

__________-[Celarent]

No minor are major

Axiom Celarent : forall major minor middle,
   holds (No middle are major)
   \ holds (All minor are middle)
   -> holds (No minor are major).

Axiom (Darii). For all terms minor, middle, and major, if All middle are major holds, and Some minor are middle holds, then Some minor are major also holds.

All middle are major  Some minor are middle

__________-[Darii]

Some minor are major

Axiom Darii : forall major minor middle,
   holds (All middle are major)
   \ holds (Some minor are middle)
   -> holds (Some minor are major).

Closing the Module

End TraditionalLogicCheat.